

Nash equilibrium with mark-up-pricing oligopolists

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Abstract

If business managers formulate strategies in terms of mark-ups, then it is natural to think they will think of their rivals' actions in these terms. For a market characterized by constant elasticity of demand and supply, the mark-up equilibrium is derived and compared with the traditional monopoly, Cournot and perfectly competitive equilibria. We also compute the 'revenue as strategy' equilibria.

JEL classification: L10, L13

1. Introduction

Economist: As a profit-maximizing manager, you set price equal to marginal cost.

Manager: I have no idea what marginal cost is. I just set my price by adding a mark-up to my average costs. I don't worry about maximizing anything.

Economist: But what happens if demand for your product weakens?

Manager: I shade my mark-up down.

Economist (scenting victory): And what if demand goes up?

Manager (checking that no consumer advocates are within earshot): I boost my margins a bit.

Economist: Aha! So you do maximize profits, after all!

This scene, or something like it, has been played out many times since Hall and Hitch (1951) first interviewed businessmen and found that most of them claimed to follow some form of mark-up pricing. This discovery ignited a fierce methodological debate. Friedman (1953) invoked the metaphor of the expert billiards player, implicitly solving complex differential equations, to argue that businessmen could behave as profit-maximizers even if they thought in terms of rule-of-thumb mark-ups.

The idea that we do not need to know how managers perceive their choice of strategy in order to model their behavior has a strong appeal in the analysis of competitive markets. Competition in

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product markets and in the market for corporate control will ensure that managers who do not set price equal to marginal cost will ultimately be driven from the market.

This argument, however, does not extend to oligopolistic markets. Even if managers act so as to maximize their own profits, they must formulate some view about the way their rivals will act. If business managers formulate strategies in terms of mark-ups, then it is natural to suppose that they will think of their rivals' actions in terms of mark-ups rather than, say, in terms of prices or quantities. If strategy is conceived in terms of mark-ups, the usual Cournot and Bertrand equilibrium concepts are inapplicable. It is necessary instead to derive a notion of Nash equilibrium where each firm's strategy set consists of a range of possible mark-ups.

In this paper we illustrate for a simply specified oligopolistic market, the difference in the Nash equilibrium outcome that results from varying the way business managers formulate their strategies. In particular we show that competition in mark-ups leads to an equilibrium outcome that is closer to the perfectly competitive outcome than the familiar Cournot outcome, while competition in revenues leads to an outcome that is closer to the joint profit-maximizing monopoly outcome.

2. Market specification

Let us consider an industry characterized by a market demand, $Q(p)$, and n identical firms. Let $Q(p) = p^{-\phi}$ (i.e. the [price] elasticity of demand is constant and equal to $\phi > 1$). Each firm can produce a quantity q at cost $c(q) = n^{1/\gamma}[\gamma/(1+\gamma)]q^{1+(1/\gamma)}$ (i.e. $\gamma > 0$ is the [constant] elasticity of market supply). Let us first consider the outcomes that may be sustained as a result of the more familiar oligopoly equilibrium notions. We shall only consider symmetric equilibrium outcomes with p and Q referring to the (equilibrium) market price and quantity, respectively, which then implies that each firm is producing Q/n .

One polar case is the perfectly competitive outcome where each firm takes the market price as given. It is characterized by the relationship that price equals marginal cost for each firm. With n symmetric firms, that corresponds to $p(Q) = c'(Q/n)$. Hence $Q^c = 1$ and $P^c = 1$.¹

The other polar case corresponds to the situation where the n firms collude to maximize their joint profit. The cost function for this industry cartel is $c^m(Q) = nc(Q/n) = [\gamma/(1+\gamma)]Q^{(1+\gamma)/\gamma}$. Hence joint profits are maximized when the industry cartel produces an amount that equates marginal revenue for the cartel with its marginal cost, i.e. $(1 - (1/\phi))(Q^m)^{-1/\phi} = (Q^m)^{1/\gamma}$, or $Q^m = (1 - 1/\phi)^{\gamma\phi/(\gamma+\phi)}$, which commands a market price $p^m = (1 - 1/\phi)^{-\gamma/(\gamma+\phi)}$.

3. Cournot (quantity competition) case

Undoubtedly the most widely applied strategy specification for a one-shot market game such as this is the Cournot–Nash quantity competition game where firms are assumed to choose their production levels taking as given what they believe their competitors are producing. The price that they anticipate receiving for their output is the price that clears the market for the total industry output that is supplied to the market.

¹ Given the normalization of demand and supply conditions in this market, the competitive outcome is invariant to changes in the parameters of the model, namely the demand and supply elasticities and the number of firms operating.

Thus firm i 's profit for producing q_i when its competitors produce in total $Q_{-i} \equiv \sum_{k \neq i} q_k$ is given by

$$\Pi^i(q_i, Q_{-i}) = (q_i + Q_{-i})^{-\frac{1}{\phi}} q_i - n^{\frac{1}{\gamma}} \frac{\gamma}{(1 + \gamma)} q_i^{\frac{1+\gamma}{\gamma}}. \quad (1)$$

Differentiating (1) with respect to q_i we obtain the FOC that implicitly defines firm i 's best (output) response to the output decisions of its competitors:

$$-\frac{1}{\phi} (q_i + Q_{-i})^{-\frac{(1+\phi)}{\phi}} q_i + (q_i + Q_{-i})^{-\frac{1}{\phi}} - n^{\frac{1}{\gamma}} q_i^{\frac{1}{\gamma}} = 0. \quad (2)$$

In a symmetric equilibrium, $q_i = Q^{\text{cn}}/n$ and $Q_{-i} = [n/(n-1)]Q^{\text{cn}}$. So (2) becomes

$$-\frac{1}{n\phi} Q^{-\frac{1}{\phi}} + Q^{-\frac{1}{\phi}} - Q^{\frac{1}{\gamma}} = 0.$$

Hence

$$Q^{\text{cn}} = \left(1 - \frac{1}{n\phi}\right)^{\frac{\gamma\phi}{\gamma+\phi}} \quad \text{and} \quad p^{\text{cn}} = \left(1 - \frac{1}{n\phi}\right)^{-\frac{\gamma}{\gamma+\phi}}. \quad (3)$$

One appealing comparative static feature of this outcome is that as the market becomes more 'competitive' (either by increasing n , the number of firms operating, or ϕ , the price elasticity of demand) the Cournot–Nash outcome converges to the perfectly competitive case. However, in the next two sections we demonstrate that alternative formulations for business managers' strategies result in equilibrium outcomes that are just as well defined and well behaved but distinct from the above Cournot–Nash equilibrium outcome.

4. Competition in (average) mark-ups

Let $[\theta_1, \dots, \theta_n]$ be the vector of mark-ups (average profit levels) of the n firms. Given the above market characteristics, these mark-ups will lead to vector of production levels $[q_1, \dots, q_n]$ for the n firms, a market price p and market quantity traded and sold of $Q = \sum_k q_k$. By definition we have for each firm i :

$$\frac{c(q_i)/q_i}{p(Q)} = \frac{1}{1 + \theta_i} \equiv \lambda_i. \quad (4)$$

From Eq. (4) it follows that $q_i/q_j = (\lambda_i/\lambda_j)^\gamma$ and so

$$q_i = \frac{\lambda_i^\gamma}{\sum_k \lambda_k^\gamma} Q. \quad (5)$$

Substituting (5) back into (4) we obtain:

² The reader can verify that for this cost structure, average cost is always a constant fraction of marginal cost; hence it readily follows in this case that competition in mark-ups over average cost is strategically equivalent to competition in mark-ups over marginal cost.

$$Q = \left[\left(\frac{\sum_k \lambda_k^\gamma}{n} \right)^{\frac{1}{\gamma}} \left(\frac{1+\gamma}{\gamma} \right) \right]^{\frac{\gamma\phi}{\gamma+\phi}}, \quad (6)$$

$$p = \left[\left(\frac{\sum_k \lambda_k^\gamma}{n} \right)^{\frac{1}{\gamma}} \left(\frac{1+\gamma}{\gamma} \right) \right]^{-\frac{\gamma\phi}{\gamma+\phi}}. \quad (7)$$

Letting $\Pi^i(\lambda_i, \lambda_{-i})$ denote the profit earned by firm 1 when it selects average profit rate $\lambda_i^{-1} - 1$ and its competitors select average profit rate corresponding to $\lambda_{-i} \equiv [\lambda_1, \dots, \lambda_{i-1}, \lambda_{i+1}, \dots, \lambda_n]$ we have

$$\Pi^i(\lambda_i, \lambda_{-i}) = R^i(\lambda_i, \lambda_{-i}) - C^i(\lambda_i, \lambda_{-i}), \quad (8)$$

where

$$R^i(\lambda_i, \lambda_{-i}) = pq_i = \lambda_i^\gamma \left[\sum_k \lambda_k^\gamma \right]^{-\frac{(\gamma+1)}{\gamma+\phi}} \left[n^{-\frac{1}{\gamma}} \left(\frac{1+\gamma}{\gamma} \right) \right]^{\frac{\gamma(\phi-1)}{\gamma+\phi}}$$

and

$$C^i(\lambda_i, \lambda_{-i}) = n^{-\frac{1}{\gamma}} \left(\frac{1+\gamma}{\gamma} \right)^{\frac{1+\gamma}{\gamma}} q_i^{\frac{1+\gamma}{\gamma}} = \lambda_i^{1+\gamma} \left[\sum_k \lambda_k^\gamma \right]^{-\frac{(\gamma+1)}{\gamma+\phi}} \left[n^{-\frac{1}{\gamma}} \left(\frac{1+\gamma}{\gamma} \right) \right]^{\frac{(\gamma-1)(\phi-1)}{\gamma+\phi}}.$$

In a symmetric equilibrium $\lambda_i = \lambda^{\text{mu}}$, for $i = 1, \dots, n$ and $\theta^{\text{mu}} \equiv (\lambda^{\text{mu}})^{-1} - 1$, is the profit-maximizing (average) mark-up for firm i , given that all its competitors are also choosing mark-ups of θ^{mu} . Hence it follows that for the symmetric equilibrium mark-up profile:

$$\frac{\frac{\partial \ln R^i(\lambda_i, \lambda_{-i})}{\partial \ln \lambda_i}}{\frac{\partial \ln C^i(\lambda_i, \lambda_{-i})}{\partial \ln \lambda_i}} = \frac{(\partial R^i / \partial \lambda_i)(\lambda_i / R^i)}{(\partial C^i / \partial \lambda_i)(\lambda_i / C^i)} = \frac{C^i}{R^i} = \lambda^{\text{mu}}.$$

So

$$\lambda^{\text{mu}} = \left(\frac{\gamma}{1+\gamma} \right) \left[1 - \frac{1}{\gamma(n-1) + \phi n} \right] \quad (9)$$

and

$$\theta^{\text{mu}} = \frac{1}{\gamma} \left[1 + \frac{1+\gamma}{\gamma(n-1) + \phi n - 1} \right]. \quad (10)$$

Substituting (9) into (6) and (7) we see that

$$Q^{\text{mu}} = \left[1 - \frac{1}{\gamma(n-1) + \phi n} \right]^{\frac{\gamma\phi}{\gamma+\phi}} \quad \text{and} \quad p^{\text{mu}} = \left[1 - \frac{1}{\gamma(n-1) + \phi n} \right]^{-\frac{\gamma}{\gamma+\phi}}. \quad (11)$$

Note that although this mark-up equilibrium exhibits qualitatively similar comparative static behavior as the Cournot–Nash equilibrium with respect to $n\phi$, the product of the number of firms operating and the degree of price responsiveness of market demand, there is an additional term, $\gamma(n-1)$, that makes this equilibrium outcome more competitive than its Cournot–Nash counter-

part. Loosely speaking, if a business manager decides to reduce her mark-up, given that her competitors keep their mark-ups unchanged, the resulting increase in her production is matched by a *reduction* in her competitors' total production. Thus the price change induced by her decision to change her mark-up is less than is the case of Cournot quantity competition, leading to equilibrium behavior that is closer to that of (price-taking) perfectly competitive firms. Intuitively, the strength of this effect is stronger, the larger the quantity response a (competitor) firm requires to reduce its average cost in order to maintain its mark-up (that is, the larger is γ), and the larger the number of competitors (that is $n - 1$).

For competition in mark-ups we see that provided there are at least two firms, the equilibrium outcome converges to the competitive outcome not just for n or ϕ tending to infinity but also for γ . Of course $\gamma = \infty$ corresponds to the case of linear cost (that is, constant marginal cost) and unsurprisingly competition in mark-ups is strategically equivalent to Bertrand price competition.³

5. Revenue as a strategy

To contrast with the mark-up equilibrium, which was more competitive than the Cournot–Nash equilibrium, let us consider a strategy specification that results in an equilibrium outcome that is less competitive than the Cournot–Nash.

Let $[R_1, \dots, R_n]$ be the vector of revenue targets of the n firms. These revenue targets will lead to a vector of production levels $[q_1, \dots, q_n]$ for the n firms and a market price p and market quantity traded and sold of $Q = \sum_k q_k$ with $p q_i = R_i$. Hence

$$\left(\sum_k R_i \right) / p = Q = p^{-\phi}$$

and so

$$p = \left(\sum_k R_k \right)^{\frac{1}{1-\phi}}, \quad Q = \left(\sum_k R_k \right)^{\frac{\phi}{\phi-1}} \quad \text{and} \quad q_i = R_i \left(\sum_k R_k \right)^{\frac{1}{\phi-1}}. \quad (12)$$

Letting $\Pi^i(R_i, R_{-i})$ denote the profit earned by firm i when it selects revenue target R_i and its competitors select revenue targets $R_{-i} \equiv [R_1, \dots, R_{i-1}, R_{i+1}, \dots, R_n]$ we have

$$\Pi^i(R_i, R_{-i}) = R_i - n \frac{1}{\gamma} \left(\frac{\gamma}{1+\gamma} \right) R_i^{\frac{1+\gamma}{\gamma}} \left(\sum_k R_k \right)^{\frac{1+\gamma}{\gamma(\phi-1)}}. \quad (13)$$

Differentiating (13) and setting it to zero we obtain

$$1 - n \frac{1}{\gamma} \left[R_i^{\frac{1}{\gamma}} \left(\sum_k R_k \right)^{\frac{1+\gamma}{\gamma(\phi-1)}} + \frac{1}{(\phi-1)} R_i^{\frac{1+\gamma}{\gamma}} \left(\sum_k R_k \right)^{\frac{1+2\gamma-\gamma\phi}{\gamma(\phi-1)}} \right] = 0. \quad (14)$$

³ Note only for the case of constant marginal cost does there exist a symmetric pure strategy equilibrium in the Bertrand price competition game. We concur with Tirole's (1988, p. 215) assessment that a mixed strategy for a static price game is difficult to interpret because although it might be optimal before the firm knows the realization of the other firms' price choices, generally after learning these realizations the firm may want to change its price.

In a symmetric equilibrium $R_i = R_k = R^{\text{rev}}$ and from (14) it follows that

$$R^{\text{rev}} = \left[1 + \frac{1}{n(\phi - 1)} \right]^{-\frac{\gamma(\phi - 1)}{\gamma + \phi}} n^{-1}. \quad (15)$$

Hence from (12) it follows that

$$p^{\text{rev}} = \left[\frac{n(\phi - 1)}{n(\phi - 1) + 1} \right]^{-\frac{\gamma}{\gamma + \phi}} = \left[1 - \frac{1}{n\phi - n + 1} \right]^{-\frac{\gamma}{\gamma + \phi}} \quad (16)$$

and

$$Q^{\text{rev}} = \left[1 - \frac{1}{n\phi - n + 1} \right]^{\frac{\gamma\phi}{\gamma + \phi}}. \quad (17)$$

The reason that this outcome is less competitive than the Cournot–Nash outcome is, speaking loosely again, that if a business manager decides to increase his revenue target, given that his competitors keep their revenue targets unchanged, the resulting increase in his production is matched by an *increase* in his competitors' total production. Thus the price change induced by his decision to change his revenue strategy is greater than is the case of Cournot quantity competition, leading to equilibrium behavior that is closer to that of a joint profit-maximizing cartel. Intuitively, the strength of this effect is stronger, the larger the number of competitors (that is, $n - 1$).

6. Conclusion

If strategy is conceived of in terms of mark-ups, the usual Cournot and Bertrand equilibrium concepts are inapplicable. It is necessary instead to derive a notion of Nash equilibrium where each firm's strategy set consists of a range of possible mark-ups. The Cournot–Nash equilibrium concept for one-shot games has been very widely applied. Other possible specifications of the strategy space yield, however, equilibria more and less competitive than Cournot. This indeterminacy gives emphasis to the need to justify the choice of a strategy space to represent the interactions between firms as a one-shot game.

One line of justification for the use of the Cournot strategy space is that it represents the reduced form of a two-stage game in which firms first commit themselves to a given capacity and then compete on price [see Kreps and Scheinkman (1983)]. Similarly, the mark-up strategy space used here might be justified as the outcome of an evolutionary process in which firms begin by following naive mark-up strategies. Over time those mark-up strategies yielding suboptimal profits will be selected out, and the Nash equilibrium will emerge. Neither of these stories will fit all markets, and it is therefore unlikely that any single strategy specification and its associated Nash equilibrium outcome(s) will be appropriate for oligopolistic markets in general. Therefore, if such markets are to be modelled as one-shot games, it is important to give careful consideration to the appropriate representation of the strategy space, rather than arbitrarily selecting, say, the Cournot equilibrium.

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