

THE FISCAL DIVIDEND MYTH OF AN INCOME/GST TAX SWITCH*

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Reaping a windfall fiscal dividend from the taxation of the 'underground' economy's expenditures on 'legitimate' commodities is often seen as a significant advantage for a goods and services tax (GST) over an income tax. This claim ignores the changes in prices in the underground economy which would arise from the introduction of a GST. Employing a general equilibrium model which allows for tax evasion, we show that any 'dividend' arising from a change in the income tax/GST mix is equivalent to a rise in the income tax rate without a GST.

I. INTRODUCTION

What is the effect on the 'underground' economy of a change in government revenue collection from a system based solely on an income tax, to one based on both an income tax and a broad-based goods and services or value added tax (GST and VAT respectively)? Specifically, if the tax switch is designed to be revenue-neutral for honest tax-payers, is there any 'fiscal dividend' to the government through capturing part of the receipts of the underground economy when they are used to purchase goods and services from the legitimate sector of the economy?¹

Reaping a tax windfall from the underground economy has often been touted as an added benefit resulting from the introduction of a GST. Kesselman(1993) cites various government reports, political party policy documents and public finance commentators from the United States, Japan, the United Kingdom, Australia and New Zealand that espouse this view.² The logic of the argument seems eminently plausible. The increased indirect tax burden of the honest taxpayer is matched by a reduction in income tax. Conversely, while a tax-evader operating in the underground economy will evade the goods and services tax as well as the income tax on undeclared transactions, that part of her undeclared earnings that is spent on legitimate goods and services will now incur the sales tax. As a consequence, total tax revenue should rise.

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¹ A GST or a VAT is a multi-stage tax applied to the value added at each stage of the production and distribution process. In contrast, both a retail sales tax and a wholesale tax applies to a single transaction stage. This paper only deals with GST/VAT type taxes.

² See Kesselman (1993, p. 132) and references therein. See also Peacock and Shaw (1982a,b).

As Kesselman (1993) emphasises, such reasoning only provides a partial equilibrium analysis. To assess the 'fiscal dividend' claim requires a general equilibrium model. Employing a particular specification of a perfectly competitive general equilibrium model which allows for tax evasion (developed in Kesselman, 1989), he demonstrates that in the case where the underground sector can equally evade both a GST and an income tax, there are no *real* effects resulting from a revenue neutral income/GST tax switch. The same set of transactions that occurred before the switch still obtain in equilibrium after the switch. An immediate corollary is that if the tax switch had been designed to be revenue-neutral for honest taxpayers, there would be no change in (real) government revenue and no 'fiscal dividend' would be forthcoming.

This paper shows that Kesselman's fiscal neutrality result holds in considerable generality. The general equilibrium model that we develop dispenses with all of Kesselman's specific assumptions on the technology of the economy except for the 'separability' of the underground and legitimate sectors. Thus, the result does *not* depend on any specific assumptions about the nature of competition, the form of the demand functions or price-taking behaviour. In fact the model does not even require that agents maximise utility or that firms maximise profit, although both of these possibilities are encompassed by our model. Rather the model relies on the assumption of separability and a minimal behavioural assumption that rules out 'nominal (or money) illusion' for agents.

The assumption of 'separability' requires that the underground sector only uses undeclared inputs to produce undeclared outputs, while the legitimate sector only uses declared inputs. This assumption is crucial for the results presented in Section II of this paper. We discuss the implications of weakening this assumption in Section III.

To see the intuition behind the fiscal neutrality result, consider a revenue-neutral income/GST tax switch for an economy with no underground sector. As consumers have been compensated for the rise in the goods and services tax it appears reasonable to assume (at least for a first pass) that the GST is fully passed on to the consumer through higher prices. Consider now introducing an underground sector. This may involve a provider of services that are not declared to the Inland Revenue Service; say a plumber undertaking odd jobs around the neighbourhood on a casual basis. At the original fee those services provide less purchasing power for 'legitimate' goods and services that are now subject to the GST. We can imagine the maintenance of purchasing power by increasing the fee charged for these undeclared services. The recipients of those services should be quite willing and able to pay that increased fee since either they have had an income tax cut to compensate them for GST induced price increases or they have increased the price of their own undeclared services. The net effect is thus: any GST revenue generated from purchases of 'legitimate' goods and services out of undeclared income is matched by a reduction in income tax paid by purchasers of 'underground' goods and services out of declared income.

The above intuition suggests that the only change resulting from a revenue neutral income/GST tax switch is that the prices of *all* goods and services (including those in the underground sector) and the wages of workers in the underground sector experience a percentage rise equal to the rate of the GST. It is reasonably straightforward to see that these prices support exactly the same set of production and consumption decisions as before the switch since every agent's economic constraint set is unaltered in real terms. For producers in the legitimate sector all GST exclusive prices and income-tax inclusive wages are unchanged. For producers in the underground sector all prices and wages have increased by the same proportion. Finally, consumers face unchanged

relative prices and have the same real disposable income. Their disposable incomes have risen in nominal terms by the amount of the GST since any income earned from the underground sector has increased as a result of higher prices and wages in that sector and any income earned from the legitimate sector is now subject to a commensurably lower rate of income tax. By Walras' Law, government real revenue (and expenditure) is thus unaltered as well.

A corollary of the above discussion is that, where fiscal neutrality holds, any increase in real revenue arising from a change in the tax mix is equivalent to a rise in the income tax rate. That is, any 'fiscal dividend' is really a disguised tax rise.

II. A GENERAL EQUILIBRIUM ANALYSIS OF A CHANGE IN THE INCOME/CONSUMPTION TAX MIX

In this section we present a formal model of fiscal neutrality. We impose only a weak consistency axiom on the 'equilibrium' behaviour of the economic agents but, as noted above, require that legitimate and underground production is never mixed.

Consider an economy consisting of:

- a) I consumers with a generic consumer denoted by i ;
- b) J 'legitimate' firms with a generic 'legitimate' firm denoted by j ;
- c) K 'underground' firms with a generic 'underground' firm denoted by k ; and
- d) a government sector.

The government can set a (proportional) income tax rate t , and/or a (uniform) goods and services tax rate s .³ The GST creates a 'wedge' between the price that a consumer pays for a 'legitimate' good and the price received by the producer. Similarly, the income tax drives a wedge between the take-home pay of a worker and the wage paid by the producer in a 'legitimate' firm; and between the profits earned by a legitimate firm and distributed to the owners of that firm. By definition no such wedges apply to 'underground' prices, wages and profits.

Let p denote a vector of goods and services prices, w denote a wage rate and π denote a profit level. The above relationships can be summarised by

$$p_c^L = p^L(1+s), \quad p_c^U = p^U, \quad w_c^L = w^L(1-t), \quad w_c^U = w^U, \\ \pi_{j,c}^L = \pi_j^L(1-t) \quad \text{and} \quad \pi_{j,c}^U = \pi_j^U$$

where superscript L (respectively U) refers to the 'legitimate' (respectively 'underground') sectors; subscript c refers to the consumer; and without the subscript c refers to the producer.

³ To allow for comparisons between different tax rate mixes we:

- a) use a simplified tax structure involving constant marginal rates;
- b) assume full imputation for tax paid on company profits distributed as dividends to shareholders; and
- c) assume that the interest income earned on savings is tax exempt.

This will enable us to alter the tax mix so that a person who does not transact in the underground economy will face an unchanged tax burden regardless of their pattern of consumption. Clearly, the income tax scheme of no country satisfies all these assumptions. Weakening these assumptions will generally entail a tax switch having additional revenue implications within our model. However, these revenue effects will arise even in the *absence* of tax evasion and they do not revive the 'fiscal dividend' that is referred to in the introduction.

Individual i consumes a vector of 'legitimate' and 'underground' goods denoted by x_i^L and x_i^U , respectively, and supplies the quantity ℓ_i^L of labour to the legal sector, and ℓ_i^U to the underground sector. Firms are owned by consumers. Let θ_{ij} be i 's share in the 'legitimate' firm j , and θ_{ik} be i 's share in the 'underground' firm k .

Each 'legitimate' firm j operates a production plan (y_j^L, ℓ_j^L) from its set of production possibilities. Similarly, each 'underground' firm k operates a feasible production plan (y_k^U, ℓ_k^U) . Production may involve the use of intermediate goods. However, by our 'separation' assumption the legitimate and underground sectors of the economy do not interact in production. Thus, y_j^L may include negative elements, but can never involve a good produced through the underground economy. Similarly y_k^U can never involve a good produced through the legitimate economy. 'Separation' thus ensures that total spending on 'legitimate' goods and services must equal total payments to 'legitimate' factors and similarly that total spending on 'underground' goods and services must equal total payments to 'underground' factors.

The government claims a vector of output from the legitimate economy only, denoted by g .⁴ It pays for these goods through tax revenue.

A 'state' of this economy is described by a set of prices, wages, tax rates and real activities of all the economic actors. For a state of the economy to be both feasible and internally consistent we require it to satisfy the following definition.

Definition 3.1

$$\left(p^L, p^U, w^L, w^U, t, s, g, \{x_i^L, x_i^U, \ell_i^L, \ell_i^U\}_{i=1}^I, \{y_j^L, \ell_j^L\}_{j=1}^J, \{y_k^U, \ell_k^U\}_{k=1}^K \right)$$

is a consistent price allocation if:

- i) $\{y_j^L, \ell_j^L\}_{j=1}^J, \{y_k^U, \ell_k^U\}_{k=1}^K$ are feasible production plans;
- ii) for all i, j and k

$$(1+s)p^L x_i^L + p^U x_i^U = (1-t)w^L \ell_i^L + w^U \ell_i^U + \sum_j \theta_{ij} \pi_j^L (1-t) + \sum_k \theta_{ik} \pi_k^U \quad (2)$$

$$\pi_j^L = p^L y_j^L - w^L \ell_j^L \quad (3)$$

$$\pi_k^U = p^U y_k^U - w^U \ell_k^U \quad (4)$$

$$(1+s)p^L g = \sum_i t \left(w^L \ell_i^L + \sum_j \theta_{ij} \pi_j^L \right) + sp^L \sum_i x_i^L + sp^L g \quad (5)$$

⁴ We assume that the government itself does not participate in the underground economy.

$$\text{iii) } \sum_i x_i^L + g = \sum_j y_j^L, \quad \sum_i x_i^U = \sum_k y_k^U, \quad \sum_i \ell_i^L = \sum_j \ell_j^L, \quad \sum_i \ell_i^U = \sum_k \ell_k^U.$$

Equation (2) is consumer i 's budget constraint, equations (3) and (4) are definitions of pre-tax profits, and equation (5) is the government's budget constraint. Condition (iii) simply states that the total quantity purchased in any market equals the total quantity produced in that market.

Note that, a consistent price allocation does not involve any behavioural assumptions about the economic agents except that the consumers and the government are expending all (but no more) of their purchasing power.⁵

We wish to maintain a structure that is as general as possible. To this end, we neither assume any particular objective for the agents nor any specific form of strategic interaction or competition between the agents in this economy. In particular we neither assume preference maximisation for consumers, profit maximisation for producers, nor price-taking behaviour for any actor. Hence the notion of 'equilibrium' for this economy will be informally defined as a consistent price allocation from which *no economic agent chooses to deviate*.

Whatever the motivations of the agents and the manner in which they interact, we suggest that a minimal consistency requirement for their equilibrium behaviour is that they should not suffer from 'money' or 'nominal' illusion. Essentially, if an equilibrium had nominal quantities denominated in dollars and then we converted all nominal quantities into their Yen equivalents, the same set of real activities and trades with the Yen prices would still constitute an equilibrium. The assumption below incorporates this intuition and in addition allows for changes in tax rates, provided that all *relative, post-tax* prices remain unchanged.

Assumption on agents' equilibrium behaviour (No Nominal Illusion)

If $\left(p^L, p^U, w^L, w^U, t, s, g, \{x_i^L, x_i^U, \ell_i^L, \ell_i^U\}_{i=1}^I, \{y_j^L, \ell_j^L\}_{j=1}^J, \{y_k^U, \ell_k^U\}_{k=1}^K \right)$ is an equilibrium of the economy and $\left[\tilde{p}^L(1+\tilde{s}), \tilde{p}^U, \tilde{w}^L(1-\tilde{t}), \tilde{w}^U \right] = \alpha \left[p^L(1+s), p^U, w^L(1-t), w^U \right]$ with $\frac{\tilde{p}^L}{\tilde{w}^L} = \frac{p^L}{w^L}$ for some $\alpha > 0$ then $\left(\tilde{p}^L, \tilde{p}^U, \tilde{w}^L, \tilde{w}^U, \tilde{t}, \tilde{s}, g, \{x_i^L, x_i^U, \ell_i^L, \ell_i^U\}_{i=1}^I, \{y_j^L, \ell_j^L\}_{j=1}^J, \{y_k^U, \ell_k^U\}_{k=1}^K \right)$ is also an equilibrium.

⁵ If we interpret equations (2) and (5) as intertemporal budget constraints with present value prices, this assumption implies that consumers expend their lifetime purchasing power and that the government balances its intertemporal budget. Interpreting these budget constraints as intertemporal ones entails implicitly assuming that interest income from savings (and interest payments from dissavings) are not taxable (tax deductible). The removal of the double taxation of saving is a well known and analysed consequence of an indirect/direct tax switch but it is not germane to the issue of the fiscal dividend since it can be accomplished within the income tax code by exempting such interest income from savings for income taxation purposes.

We show in the appendix that not only is $(\bar{p}^L, \bar{p}^U, \bar{w}^L, \bar{w}^U, \bar{t}, \bar{s}, \bar{g}, \{x_i^L, x_i^U, \ell_i^L, \ell_i^U\}_{i=1}^I, \{y_j^L, \ell_j^L\}_{j=1}^J, \{y_k^U, \ell_k^U\}_{k=1}^K)$ a consistent price allocation but also that the real income of every economic actor is unaffected. Thus, it seems reasonable to assume that, for whatever decisions were undertaken by the economic agents' to support the original equilibrium, there exists a set of decisions for the agents, with equivalent 'real' outcomes, that supports the new equilibrium.

The assumption on agents' equilibrium behaviour is far weaker than (and encompasses) the Arrow-Debreu model of perfectly competitive markets with utility maximising consumers and profit maximising firms. In terms of the standard Arrow-Debreu general equilibrium framework, the assumption is equivalent to agent indirect utility functions and firm profit functions both being homogeneous of degree zero in all nominal variables. However, our results only depend on the weaker consistency assumption presented above which does not require explicit optimising behaviour by agents.

The assumption on agents' equilibrium behaviour will only apply if all relative prices faced by an economic agent do not alter. However, the proposition below shows that such a change is exactly what can occur if we alter the sales and income tax mix.

Initially consider the case where the government only levies an income tax, $t = \bar{t}$ (and hence $s = 0$). For these tax parameters, let

$$(\bar{p}^L, \bar{p}^U, \bar{w}^L, \bar{w}^U, \bar{t}, \bar{s} = 0, \bar{g}, \{\bar{x}_i^L, \bar{x}_i^U, \bar{\ell}_i^L, \bar{\ell}_i^U\}_{i=1}^I, \{\bar{y}_j^L, \bar{\ell}_j^L\}_{j=1}^J, \{\bar{y}_k^U, \bar{\ell}_k^U\}_{k=1}^K)$$

denote an equilibrium consistent price allocation.

Now, consider the introduction of a GST combined with a 'compensatory' reduction in the income tax rate. We wish to design a new system that will not penalise individuals who were previously not engaging in tax evasion. Thus, if we denote the new income tax rate by \hat{t} , and the new GST tax rate by \hat{s} , we require that

$$\frac{(1-\hat{t})}{(1+\hat{s})} = (1-\bar{t}) \quad (6)$$

To see that a person in the 'legitimate' economy would view this change as neutral, given the nominal pre-tax wage and price levels, note that under the original scheme, a dollar of pre-tax earnings led to $\$(1-\bar{t})$ after tax. Now, a dollar of pre-tax income leads to $\$(1-\hat{t})$ after income tax, but each dollar has had its real purchasing power reduced through the sales tax by a factor $1/(1+\hat{s})$.

The following proposition demonstrates that when we allow for 'general equilibrium' adjustments, the above change in the tax mix can leave all economic agents, including the government, unaffected in real terms.

Proposition One

For any equilibrium of the economy under the old tax regime there exists an equilibrium in the economy under the new tax regime in which there is no change in any real variables. Specifically, the government has no change in its (real) revenue and there is no real fiscal dividend.

Proof

Consider the following prices

$$\bar{p}^L = \hat{p}^L, \bar{w}^L = \hat{w}^L, \bar{p}^U(1 + \hat{s}) = \hat{p}^U \text{ and } \bar{w}^U(1 + \hat{s}) = \hat{w}^U$$

Note that, $\frac{\hat{p}^L}{\hat{w}^L} = \frac{\bar{p}^L}{\bar{w}^L}$ and from applying equation (6) we have

$$\left[\hat{p}^L(1 + \hat{s}), \hat{p}^U, \hat{w}^L(1 - \hat{t}), \hat{w}^U \right] = \alpha \left[\bar{p}^L, \bar{p}^U, \bar{w}^L(1 - \hat{t}), \bar{w}^U \right] \text{ for } \alpha = 1 + \hat{s}.$$

Hence by the *No Nominal Illusion Assumption* from above

$$\left(\hat{p}^L, \hat{p}^U, \hat{w}^L, \hat{w}^U, \hat{t}, \hat{s}, \hat{g}, \left\{ \bar{x}_i^L, \bar{x}_i^U, \bar{\ell}_i^L, \bar{\ell}_i^U \right\}_{i=1}^I, \left\{ \bar{y}_j^L, \bar{\ell}_j^L \right\}_{j=1}^J, \left\{ \bar{y}_k^U, \bar{\ell}_k^U \right\}_{k=1}^K \right)$$

is an equilibrium. ■

Proposition one shows that a tax change that is neutral for a person who operates only in the legitimate economy is also neutral with respect to all real variables. This proposition is directly analogous to Kesselman's (1993, p. 138) neutrality result when the underground economy fully avoids both income and sales tax. However, proposition one only requires minimal behavioural assumptions and significantly extends Kesselman's neutrality result. In particular, only minimal consistency of behaviour is required for proposition one.

To see, explicitly, that there is no 'fiscal dividend' notice that, the aggregate budget constraint of the legitimate sector is originally

$$\bar{p}^L \bar{x}_i^L = (1 - \hat{t}) \left[\bar{w}^L \bar{\ell}_i^L + \sum_j \theta_j \bar{\pi}_j^L \right]$$

The new tax system is designed so that

$$(1 + \hat{s}) \bar{p}^L \bar{x}_i^L = (1 - \hat{t}) \left[\bar{w}^L \bar{\ell}_i^L + \sum_j \theta_j \bar{\pi}_j^L \right]$$

Subtracting the former from the latter we obtain

$$\hat{s} (\bar{p}^L \bar{x}_i^L) = (\hat{t} - \hat{t}) \left[\bar{w}^L \bar{\ell}_i^L + \sum_j \theta_j \bar{\pi}_j^L \right]$$

The proposition demonstrates that the tax switch is consistent with no change in 'legitimate' pre-tax prices or quantities hence we have

$$\hat{s}(\hat{p}^t \hat{x}_i^t) + \hat{r} \left[\hat{w}^t \hat{z}_i^t + \sum_j \theta_{ij} \hat{\pi}_j^t \right] = \hat{r} \left[\hat{w}^t \hat{z}_i^t + \sum_j \theta_{ij} \hat{\pi}_j^t \right]$$

showing that government revenue is indeed constant under both tax systems.

The following corollary exposes the basis of the fiscal dividend myth.

Corollary to Proposition One

Consider an economy as described above which has a unique equilibrium under any given tax regime. Any change in the tax mix which raises real government revenue is equivalent to a rise in income tax on declared earnings.

The corollary follows directly from proposition one. Any fiscal regime with income tax t and goods and services tax s gives rise to an equilibrium which is identical in real terms to a regime with income tax rate t^* , such that $(1 - t)/(1 + s) = (1 - t^*)$, and a GST rate of zero.⁶ Thus, any change in government real tax revenue is equivalent to a change in income tax rates. The idea that changing the tax mix can raise government revenue by simply 'tapping' the underground economy, without increasing the burden on declared income is fallacious in the above model.

The corollary relies on there only being a unique equilibrium under the original tax regime. While proposition one shows the existence of an equilibrium after the income/sales tax switch which is identical in terms of all real variables to the original equilibrium, it cannot ensure the uniqueness of this equilibrium. If the economy has multiple equilibria under the original tax scheme then it will have equivalent multiple equilibria under the new tax regime. If the economy moves to a different equilibrium as a consequence of the change in tax policy then this may have real revenue implications. Such changes are however unrelated to the idea of a fiscal dividend and, to the degree that equilibria may be ranked in terms of real government revenue, may involve a reduction rather than a rise in real revenue.

III. EXTENSIONS OF THE MODEL

The neutrality result stated in proposition one above depends upon the separability of production processes between the legitimate and the underground economies. An individual can choose to work in both sectors, and can enjoy a share in both legitimate and underground firms' profits. The production of legitimate goods, however, can only involve the use of legitimate inputs, and similarly, the production of underground goods can only utilise inputs from other underground sources. Since we showed, however, that the introduction of a GST led to all GST-inclusive prices rising by the percentage of the GST, we can relax the separability assumption to allow a producer of underground goods to use legitimate and/or underground intermediate goods and still retain the fiscal neutrality result.

If we relax production separability further, we cannot draw general conclusions about the equilibrium revenue effects of a change in the tax mix without adding significantly more structure to the model. We can, however, investigate the 'impact' effects of a change in the tax regime.

⁶ This is a well-known result for an economy without tax evasion, for instance see Stiglitz (1988, p. 427).

Consider that production can utilise both legitimate and underground labour. A shift towards a GST with a compensatory fall in income tax rates will, at the original prices, tend to increase the relative supply of legitimate labour to underground labour and to reduce its wage relative to underground labour. Hence, a shift towards a GST may lead to a shift in production towards legally declared labour inputs.

Conversely, consider a firm which produces a mixture of both legitimate and undeclared goods. A shift towards a GST will at the original prices tend to enhance the profitability of the production of underground goods relative to legitimate goods.

Ironically, such impact effects would show up in official statistics as a *fall* in labour productivity. That is, an increase in measured labour inputs would coincide with a fall in measured output!

Even at this preliminary level, once we remove the assumption of production separability, it is unclear whether a change in tax regime will inspire or discourage the underground economy. It is not obvious under what conditions, if any, a 'fiscal dividend' will arise. In this sense, fiscal neutrality, as presented in proposition one, may provide the most useful benchmark for an investigation into the revenue effects of a shift in tax mix.

IV. CONCLUSION

In this paper we have investigated the foundations of the 'fiscal dividend'. We have shown, in a general equilibrium model with 'separated production' but minimal behavioural assumptions, that:

- i) a change in the tax regime towards a uniform GST, with a simultaneous reduction in the proportional income tax rate, which leaves an individual who participates only in the legitimate economy neither better nor worse off, will have no real effects on the economy and will not raise real government revenue;
- ii) a change in the tax mix which leads to a rise in real government revenue is equivalent to a rise in the income tax rate without a goods and services tax. We note that, even in a more general framework, which allows for productive interactions, it is not clear that a fiscal dividend exists.

Throughout this paper we have assumed that the ease of evasion and/or the probability of detection is the same for both income tax and a GST. Clearly, if this assumption is violated, then a change in the tax mix will have revenue implications not captured by our model.⁷ But we concur with Kesselman's (1993, p.133) assessment that the sectors that typically engage in income tax evasion are ones that would most easily lend themselves to sales tax evasion as well. Indeed in order for them to successfully evade the income tax, Kesselman notes that 'they also have to evade the indirect tax on their output, since honest reporting of their gross sales for the latter tax would signal to the authorities the extent of their income tax evasion.'

⁷ We thank Neville Norman for bringing this alternative notion of 'fiscal dividend' to our attention. Also see Kesselman (1993) and Boadway *et al.* (1992) for general equilibrium treatments of models with differential ability or propensity to evade different forms of taxation.

The above analysis has clear implications for countries considering altering the mix between direct and indirect taxation. There may well be administrative, efficiency or improved-compliance reasons for changing the tax mix away from an income based system towards a scheme based on a uniform goods and services tax. We have shown, however, that the belief that such a scheme will result in a (net) dividend from taxing undeclared earnings when they are spent in the legitimate sector of the economy is unfounded. Any 'dividend' is merely equivalent to a rise in income tax rates.

APPENDIX

In this appendix we show that if we change the prices, wages and tax rates of a consistent price allocation in a way that satisfies the *No Nominal Illusion* assumption but leave all real activities unchanged, then the resulting price allocation is still consistent. Furthermore, all relevant post-tax relative prices faced by each economic actor remain unchanged.

To see this, first note from the *No Nominal Illusion* relationship between the original and adjusted prices, wages and tax rates

$$\left[\tilde{p}^L(1+\tilde{s}), \tilde{w}^L(1-\tilde{t}) \right] = \alpha \left[p^L(1+s), w^L(1-t) \right] \text{ and } \frac{\tilde{p}^L}{\tilde{w}^L} = \frac{p^L}{w^L}$$

it follows that

$$\frac{(1-\tilde{t})}{(1+\tilde{s})} = \frac{(1-t)}{(1+s)} \quad (\text{A1})$$

For $\left(\tilde{p}^L, \tilde{p}^U, \tilde{w}^L, \tilde{w}^U, \tilde{t}, \tilde{s}, \tilde{g}, \{x_i^L, x_i^U, \ell_i^L, \ell_i^U\}_{i=1}^I, \{y_j^L, \ell_j^L\}_{j=1}^J, \{y_k^U, \ell_k^U\}_{k=1}^K \right)$ equation (3) becomes

$$\tilde{\pi}_j^L = \tilde{p}^L y_j^L - \tilde{w}^L \ell_j^L \quad (\text{A3})$$

So post-income-tax profits can be expressed as

$$\begin{aligned} (1-\tilde{t})\tilde{\pi}_j^L &= (1-\tilde{t})\left[\tilde{p}^L y_j^L - \tilde{w}^L \ell_j^L\right] \\ &= \frac{(1-\tilde{t})}{(1+\tilde{s})} \left[\alpha(1+s)p^L y_j^L\right] - \alpha(1-t)w^L \ell_j^L \quad \left[\text{since } \alpha p^L(1+s) = \tilde{p}^L(1+\tilde{s})\right] \\ &= \frac{(1-t)}{(1+s)} \left[\alpha(1+s)p^L y_j^L\right] - \alpha(1-t)w^L \ell_j^L \quad \left[\text{applying equation (A1)}\right] \\ &= \alpha(1-t)\pi_j^L \end{aligned}$$

Hence post-tax real profits for 'legitimate' firms are unchanged.

Similarly, equation (4) becomes

$$\tilde{\pi}_k^U = \tilde{p}^U y_k^U - \tilde{w}^U \ell_k^U \quad (\text{A4})$$

Hence $\tilde{\pi}_k^U = \alpha [p^U y_k^U - w^U \ell_k^U] = \alpha \pi_k^U$ showing that post-tax real profits for 'underground' firms are also unchanged.

Turning to consumer i , we have

$$\begin{aligned} (1 + \bar{s})\tilde{p}^L x_i^L + \tilde{p}^U x_i^U &= \alpha [(1 + s)p^L x_i^L + p^U x_i^U] \\ &= \alpha \left[(1 - t)w^L \ell_i^L + w^U \ell_i^U + \sum_j \theta_j \pi_j^L (1 - t) + \sum_k \theta_k \pi_k^U \right] \\ &= (1 - \bar{t})\tilde{w}^L \ell_i^L + \tilde{w}^U \ell_i^U + \sum_j \theta_j \tilde{\pi}_j^L (1 - \bar{t}) + \sum_k \theta_k \tilde{\pi}_k^U \end{aligned}$$

The above demonstrates that with the same real activities (that is consumption levels, and labour supplies) each consumer is still expending all (but no more) of their purchasing power. Moreover, note that no relative price faced by a consumer has changed.

Finally consider the government. From the market clearing condition (iii) and equation (3) it follows that

$$\begin{aligned} p^L \sum_i x_i^L + p^L g &= \sum_i \left(w^L \ell_i^L + \sum_j \theta_j \pi_j^L \right) \\ \tilde{p}^L \sum_i x_i^L + \tilde{p}^L g &= \sum_i \left(w^L \ell_i^L + \sum_j \theta_j \tilde{\pi}_j^L \right) \end{aligned}$$

and from equation (A1) it follows that $\frac{(\bar{t} + \bar{s})}{(1 + \bar{s})} = \frac{(t + s)}{(1 + s)}$.

Substituting into equation (5), we obtain

$$\begin{aligned} (1 + s)p^L g &= \sum_i (t + s) \left(p^L \sum_i x_i^L + p^L g \right) \\ \Rightarrow (1 + \bar{s})\tilde{p}^L g &= \sum_i (\bar{t} + \bar{s}) \left(\tilde{p}^L \sum_i x_i^L + \tilde{p}^L g \right) \end{aligned}$$

Multiplying both sides by $\alpha(1 + s)/(1 + \bar{s})$ and substituting $(1 + \bar{s})\tilde{p}^L$ for $\alpha(1 + s)p^L$

$$\begin{aligned} \Rightarrow (1 + \bar{s})\tilde{p}^L g &= \sum_i (\bar{t} + \bar{s}) \left(\tilde{p}^L \sum_i x_i^L + \tilde{p}^L g \right) \\ &= \sum_i \bar{t} \left(\tilde{w}^L \ell_i^L + \sum_j \theta_j \tilde{\pi}_j^L \right) + \bar{s} \left(\tilde{p}^L \sum_i x_i^L + \tilde{p}^L g \right). \end{aligned}$$

Hence the government budget constraint is still satisfied and all relative prices faced by the government are unchanged.

REFERENCES

- Boadway, R., Marchand, M. and Pestieau, P. (1994), 'Towards a Theory of the Direct-Indirect Tax Mix', *Journal of Public Economics*, vol. 55, pp. 71-88.
- Kesselman, J.R. (1989), 'Income Tax Evasion: An Intersectoral Analysis', *Journal of Public Economics*, vol. 38, pp. 137-82.
- Kesselman, J.R. (1993), 'Evasion Effects of Changing the Tax Mix', *Economic Record*, vol. 69, pp. 131-48.
- Norman, N. (1985), *The Economics of Personal Tax Escalation in Australia* (Sydney: George Allen and Unwin).
- Peacock, A. and Shaw, G.K. (1982a), 'Is Tax Revenue Loss Overstated?' *Journal of Economic Affairs*, vol. 2, pp. 161-163.
- Peacock, A. and Shaw, G.K. (1982b), 'Tax Evasion and Tax Revenue Loss', *Public Finance*, vol. 37, pp. 269-278.
- Stiglitz, J.E. (1988), *Economics of the Public Sector*, 2nd edition (New York: W.W. Norton and Company).
- Swan, P.L. (1984), 'Reforming the System: An Economist's View', in D.J. Collins (ed.), *Tax Avoidance and the Economy* (Sydney: Australian Tax Research Foundation).

