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STRATEGIC TRADE POLICY UNDER UNCERTAINTY: SUFFICIENT CONDITIONS FOR THE OPTIMALITY OF AD VALOREM, SPECIFIC AND QUADRATIC TRADE TAXES*

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In most previous work on strategic trade policy the form of government intervention has been prescribed in advance. In this paper, we apply a solution concept discussed by Klemperer and Meyer for games in which the strategy space consists of the class of all (non state-contingent) price quantity schedules. We examine a series of specific assumptions on demand and supply conditions and derive the associated equilibrium trade policies. We derive welfare implications for all cases examined.

1. INTRODUCTION

The idea that it may be desirable, with respect to the national welfare of a single country, to impose an export tax (or to achieve a similar effect through tariffs on imports) in markets where export demand is not perfectly elastic is an old one. A number of arguments have been raised against such export taxes, of which the most important is the possibility of retaliation. This raises strategic issues that have not apparently been addressed formally, but are obviously analogous to the problems of oligopoly theory. More recently, a large literature on strategic trade policy has developed (Brander and Spencer 1985, Eaton and Grossman 1986, Cooper and Riezman 1989, Laussel 1992). In this literature, it is assumed that the market is inherently oligopolistic. Governments intervene to promote the interests of their national firms, and, in some cases, the interests of domestic consumers. The normal structure is a two-stage game. In the first stage, governments commit to their intervention policies, thereby modifying the structure of the market game. In the second stage, the market game is played out between the firms.

The strategic trade literature has attracted a considerable amount of attention (positive and negative), since it appears to rationalize a number of policies actually followed by governments, such as subsidization of exports. However, it yields few clear prescriptions for the optimal form of intervention. Cooper and Riezman examine the choice between tax-subsidies and quantity-based measures, but a much broader range of policies may be considered.

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A more serious difficulty with the strategic trade literature is that the equilibrium solution and the associated policy prescription appear to depend entirely on the choice of the strategy space for firms. For example Brander and Spencer (1985), assuming Cournot competition (that is, firms compete in quantities), reach the conclusion that export subsidies may be optimal. Eaton and Grossman (1986) show that this conclusion is reversed in the case of Bertrand (price) competition. By analogy with recent results in oligopoly theory (Klemperer and Meyer 1989, and Grant and Quiggin 1994; hereafter KM and GQ, respectively), these results suggest that any possible government policy may be derived as the equilibrium of a strategic trade game, given the appropriate assumptions concerning the strategy space.

In an attempt to overcome the problem of a multiplicity of equilibria in oligopoly theory, KM propose a new equilibrium concept for a class of games under uncertainty. In these games, the strategy space consists of the set of all (non-state-contingent) price-quantity schedules. In the absence of uncertainty, KM show that with such a rich strategy space any individually rational outcome for firms can be supported as an equilibrium in price-quantity schedules. However, introducing uncertainty into the model, KM show that the set of equilibria is reduced to a connected set of price-quantity schedules. In problems of practical interest, there usually appears to exist a unique equilibrium (KM, GQ).

The KM approach has been applied to the problem of strategic trade policy by Laussel (1992). In Laussel's model the domestic and foreign firm both have identical constant marginal costs. Hence, without government intervention, using the KM solution concept, the market game would result in both firms choosing perfectly elastic price-quantity schedules where price is set at marginal cost and that earn zero profit. Laussel considers the situation where the domestic government imposes a quadratic export tax and linear subsidy. The quadratic tax steepens the slope of the tax inclusive marginal cost schedule facing the domestic firm, inducing it to choose a less price responsive price-quantity schedule in the ensuing market game. This in turn leads to a less "aggressive" choice of a price-quantity schedule by its foreign competitor and hence positive aggregate profits. The subsidy enables the domestic firm to capture a larger share of the expected profits.

In this paper, we return to the older problem of optimal intervention in the case where individual firms have no market power, but where the small country assumption does not apply. We assume that governments play a game in which they choose export and/or import tax schedules and firms act as competitive price takers. Johnson (1953) analyzed the noncooperative Nash equilibrium for the case where countries restrict trade through the use of tariffs (a proportional tax schedule in the case of an ad valorem tariff and a constant per quantity tax schedule in the case of a specific tariff). Rodriguez (1974) and Tower (1975) noted that the noncooperative Nash equilibrium outcome was markedly different if the instrument of trade policy was quotas (which can be interpreted as a nonlinear tax with zero marginal tax rate up to the quota and infinite beyond). In fact in the latter case for two countries, trade is reduced to zero.[2]

[2] We thank Richard Cornes for bringing these references to our attention.

A primary purpose of this paper is to demonstrate how the KM solution concept can be employed *to endogenize* the form of trade restriction employed by the competing governments. That is, the form of the equilibrium trade policy (whether it is a specific, ad valorem or quadratic trade tax) results from the underlying economic primitives of the model, such as the preferences of the consumers, the technology of the firms and the endowments of the countries, rather than being prescribed in advance by the analyst through the specification of the strategy space for the trade game.

We begin by clarifying the nature of equilibrium in the KM solution concept for this problem. We show that the KM solution concept has the property of ex post optimality if and only if the market price is fully revealing of the residual demand curves faced by each country. This condition will be satisfied when there exists a scalar-sufficient-statistic for the market impact of the shocks to other market participants.

With these preliminaries in hand, we examine a series of specific assumptions on demand and supply conditions and derive the associated equilibrium trade policies. For the bilateral monopoly case, we show that semi-log-linear import demand and supply curves generate an equilibrium with specific tariffs and export taxes, while constant elasticity of import and export demand curves generate an equilibrium in which ad valorem export taxes and tariffs are applied. We next analyze multi-country cases. For the case of linear export supply schedules (where negative exports are naturally identified as imports), we show that the equilibrium involves the application of quadratic trade taxes.

Some welfare implications are derived for all the cases we examine. The solution to the market game always involves a reduction in aggregate welfare relative to the competitive equilibrium. However, the loss will not in general be evenly distributed and some countries may gain as Johnson (1953) noted for the noncooperative tariff game. We derive conditions under which both countries will lose and hence under which a binding commitment to free trade would be preferred.

2. EXPORT SUPPLY (IMPORT DEMAND) FUNCTION EQUILIBRIUM WITH UNCERTAINTY

In our partial equilibrium setting the market for this tradable good is described by the competitive (that is, price-taking) export supply functions (or equivalently, the competitive import demand functions) of the n countries that potentially may consume and/or produce this good.[3] For each country i, its competitive export supply function also depends on a shock. In an analogous fashion to KM we make the following two assumptions about the countries' shocks.

ASSUMPTION 1. *Each country i's shock can be described by a* scalar *random variable $\tilde{\theta}^i$ that can take on values in* Θ^i $(=[0,\infty))$.

ASSUMPTION 2. *The joint distribution of* $(\tilde{\theta}^1,\dots,\tilde{\theta}^n)$ *has positive density on the support* $\prod_{i=1}^n \Theta^i$.

[3] Our analysis can also be interpreted as 'general equilibrium' for a two-good world.

Let $X^i(p, \theta^i)$ (respectively, $M^i(p, \theta^i)$) denote the quantity of the good that would be exported (respectively, imported) by country i if it acted perfectly competitively when the world price of that good is p and the realization $\tilde{\theta}^i$ is θ^i.

ASSUMPTION 3. *For each country i, its competitive export supply function $X^i(\cdot, \cdot)$ satisfies for all (p, θ^i), $0 < X^i_p < \infty$, $X_\theta > 0$.*

By definition, $M^i(p, \theta^i) \equiv -X^i(p, \theta^i)$. We assume that each shock θ^i is initially revealed only to country i. Upon learning of its own shock but before the shocks of the other countries are revealed to it, each country through the use of a trade tax policy commits itself to a *strategic* export-supply schedule that specifies the quantity it will export as a function of the world price (with negative quantities interpreted as demand for imports). The trade tax policy of country i implicitly embodied in its strategic export supply schedule $\hat{X}^i(\cdot, \theta^i)$ can be constructed as follows. For each quantity x^i of exports (negative if imports), the per unit tax levied by country i, is simply the difference between the world price, p, at which country i offers x^i onto the world market, and the domestic price, q^i, that would bring forth x^i of exports from the (competitive) firms in country i. That is, the tax revenue function is given by, $T^i(x^i, \theta^i) = (p - q^i)x^i$, where p and q^i are defined implicitly by $x^i = X^i(q^i, \theta^i) = \hat{X}^i(p, \theta^i)$.

After all the shocks are revealed, the market-clearing world price, $\hat{p}$ is determined with each country supplying (or demanding if it is negative) the quantity on its chosen strategic export supply schedule which intersects its realized residual demand schedule at that market-clearing world price.

It is crucial to note that each country i is unable to make its quantity of trade contingent on the shocks of the other countries (except to the extent that these affect the equilibrium price). Suppose that country i commits to a given strategy $\hat{X}^i(\cdot, \theta^i)$. Now consider a downward shift in the residual demand curve facing country i resulting in a decline in the equilibrium price from p_0 to p_1. Then, provided that the strategy $\hat{X}^i(\cdot, \theta^i)$ remains unchanged, country i will reduce output from $\hat{X}^i(p_0, \theta^i)$ to $\hat{X}^i(p_1, \theta^i)$ regardless of whether the shift in its residual demand is caused by an adverse shock in the other countries or by the adoption of a more aggressive strategy by some country j. This crucial feature of the KM equilibrium concept will be referred to as the *state-independence* assumption.[4]

The welfare that each country reaps from the resulting equilibrium is taken to be the [net] 'producer surplus' plus that country's government's [net] revenue from any tax or subsidy implicit in the chosen export supply schedule. (Of course, for a country that is importing the good the [net] 'producer surplus' might more naturally be thought of as the [net] 'consumer surplus'). This partial equilibrium welfare measure can be justified if we assume that the agents in each country can be

[4] For the specific examples considered in this paper we show that in the equilibrium in which all other countries have chosen supply schedules that are not contingent on other countries realized shocks, each country would not want to deviate unilaterally from its chosen supply schedule even if it were permitted to do so after observing the realizations of the shocks experienced by the other countries.

collectively modelled by a representative consumer with preferences that are quasi-linear with respect to the numeraire good.

Given that on learning the realization of its own shock each country j has committed itself to the export supply schedule $\hat{X}^j(\cdot, \theta^j)$, then country i's welfare can be calculated as

$$(1) \quad W^i\left(\left\langle \hat{X}^j(\cdot, \theta^j)\right\rangle_{j=1}^{n}, \theta^i\right)$$

$$= \int_0^{q^i} \max\{X^i(p, \theta^i), 0\}\, dp + \int_{q^i}^{\infty} \max\{-X^i(p, \theta^i), 0\}\, dp + (\hat{p} - q^i) X^i(q^i, \theta^i)$$

$$\text{where } \hat{p} \text{ and } q^i \text{ are defined by } \sum_{j=1}^{n} \hat{X}^j(\hat{p}, \theta^j) = 0 \,\&\, \hat{X}^i(\hat{p}, \theta^i) = X^i(q^i, \theta^i)$$

The three terms on the righthand side of equation 1 can be readily interpreted as the 'producer surplus' derived from the export of $X^i(q^i, \theta^i)(= \hat{X}^i(\hat{p}, \theta^i))$ units (nonzero if $X^i(q^i, \theta^i) > 0$); the 'consumer surplus' obtained from the importation of $-X^i(q^i, \theta^i)$ units (nonzero if $X^i(q^i, \theta^i) < 0$); and the tax revenue obtained from the trade of $X^i(q^i, \theta^i)$ units. Notice that, the third term is positive if $X^i(q^i, \theta^i) < 0$ (the country is importing the good) and the domestic price is greater than the world price, signifying a negative import subsidy, or, in other words, a positive tariff.

For completeness, we should note that following KM we shall assume that country i's welfare is zero if either no world market-clearing price exists or the quantity required to be exported by country i in equilibrium is not compatible with its underlying technology and tastes (that is, q^i fails to exist).[5]

As this is a game of imperfect information, a natural equilibrium concept to employ is Bayesian-Nash which can be shown to exist under quite general conditions. If $\langle \hat{X}^j(\cdot, \cdot)\rangle_{j=1}^{n}$ is a Bayesian-Nash equilibrium, country i adopts the following reasoning: "for each realization of my shock, θ^i, $\hat{X}^i(\cdot, \theta^i)$ is a best response to the strategy profile $\langle \hat{X}^j(\cdot, \cdot)\rangle_{j=1, j\neq i}^{n}$ given that I know only the conditional distribution of the other countries' shocks." However, following KM, we shall use the stronger concept of ex post optimal Nash equilibrium. If $\langle X^{*j}(\cdot, \cdot)\rangle_{j=1}^{n}$ is an ex post optimal Nash equilibrium, then for country i it follows that given a particular realization of $(\theta^1, \ldots, \theta^n)$, for country i, $(p, X^{*i}(p, \theta^i))$ is the welfare-maximizing price-quantity on the associated residual demand schedule that it faces. Formally:

Definition. $\langle X^{*i}(\cdot, \cdot)\rangle_{i=1}^{n}$, *is an* ex post optimal Nash equilibrium in export supply functions *if for each i and for all* $(\theta^1, \ldots, \theta^n) \in \prod_{i=1}^{n} \Theta^i$

$$(2) \quad X^{*i}(\cdot, \theta^i) \in \arg \max_{\langle \hat{X}^i(\cdot, \theta^i)\rangle} W^i\Big(\Big\langle X^{*1}(\cdot, \theta^1), \ldots, X^{*i-l}(\cdot, \theta^{j-1}), \hat{X}^i(\cdot, \theta^i),$$

$$X^{*i+1}(\cdot, \theta^{j+1}), \ldots, X^{*n}(\cdot, \theta^n)\Big\rangle, \theta^i\Big)$$

[5] For the particular specifications that follow in subsequent sections, this point will be ignored as it will not form part of any country's equilibrium behavior.

Ex post optimality will arise if the world market-clearing price that arises for each realization of $(\theta^1, \dots, \theta^n)$, reveals fully to country i the residual demand that it faces conditional on its own shock θ^i. This condition is trivially satisfied when there are only two countries, each facing a scalar shock. More generally, it will be satisfied if, given the equilibrium profile of strategic supply-schedules, the effect on residual demand for each country i of the shocks to all other countries may be summarized by a scalar random variable $\tilde{\psi}^i$. The equilibria of the particular examples that we analyze in the sections below will all be shown to satisfy this property.

For a profile of (strategic) 'export' supply functions $\langle X^{*i}(\cdot,\cdot)\rangle_{i=1}^n$ let $p^*(\theta)$ and $q^{*1}(\theta), \dots, q^{*n}(\theta)$ be the world market-clearing price, and n domestic prices defined as in (1) above for the profile of realizations of country shocks θ in $\prod_{i=1}^n \Theta^i$. If country i is an exporter in the equilibrium for this profile of realizations of country shocks, it follows that for $(p^*(\theta), X^{*i}(p^*(\theta), \theta^i))$ to be the (ex post) welfare-maximizing price-quantity combination for country i, the marginal gain to country i from exporting the last unit matches the loss it incurs on all inframarginal exports as a result of the reduction in the world price necessary to accommodate that last unit of exports. The former is simply the difference between the world price received for that unit, that is $p^*(\theta)$, and country i's opportunity cost of making it available to the world market, that is the domestic price $q^{*i}(\theta)$.[6] The latter is simply the quantity of exports $X^{*i}(p(\theta), \theta^i)$ $(= -\Sigma_{j\neq i} X^{*j}(p, \theta^j))$ multiplied by the associated reduction in the world price, that is $(\Sigma_{j\neq i}\, \partial X^{*j}(p, \theta^j)/\partial p)^{-1}$. Analogous reasoning applies for a country who is an importer in equilibrium for a particular profile of realizations of country shocks (except with the appropriate switching of signs). Hence it follows that a necessary condition for a profile of differentiable export supply functions $\langle X^{*i}(\cdot,\cdot)\rangle_{i=1}^n$ to be an (ex post optimal Nash) equilibrium in export supply functions is that for all i and all $\theta \in \prod_{i=1}^n \Theta^i$

$$p^*(\theta) - q^{*i}(\theta) = \frac{-\Sigma_{j\neq i} X^{*j}(p, \theta^j)}{\Sigma_{j\neq i}\, \partial X^{*j}(p, \theta^j)/\partial p}$$

or if we divide both sides by $p^*(\theta)$ we obtain the well-known "inverse-elasticity" rule

(3)

$$\frac{p^*(\theta) - q^{*i}(\theta)}{p^*(\theta)} = \frac{1}{\varepsilon^{*i}(p^*(\theta), \theta)} \text{ where } \varepsilon^{*i}(p, \theta) \equiv -\frac{\Sigma_{j\neq i}\, \partial X^{*j}(p, \theta^j)/\partial p}{\Sigma_{j\neq i} X^{*j}(p, \theta^j)/p}$$

Equation 3 is closely related to the well-known Lerner index that states that a profit-maximizing firm chooses a price-quantity point on the (residual) demand schedule that it faces where the price-(marginal) cost margin is equated with the inverse of the (absolute value of the) price elasticity of (residual) demand. For an exporting country i the residual demand that it faces is $-\Sigma_{j\neq i} X^{*j}(p, \theta^j)$ for which the price elasticity is indeed $\varepsilon^{*i}(p, \theta)$. The marginal (or opportunity) cost is the

[6] Notice that the domestic price $q^i(\theta)$ measures both the valuation of the marginal unit consumed and the resource opportunity cost of the marginal unit produced in country i.

domestic price $q^i(\theta)$ that measures both the valuation of the marginal unit consumed and the resource opportunity cost of the marginal unit produced in that country. Intuitively, the optimal supply schedule involves an implicit export tax that restricts the exporting country's supply, leading to an increase in the world price that it receives for those units it does export. That is, it exploits its monopoly power with respect to the residual demand that it faces. Conversely for an importing country, $-\sum_{j\neq i} X^{*j}(p,\theta^j)$ is negative, and the optimal trade policy involves an implicit import tax (i.e. tariff) with the domestic price $q^{*i}(\theta)$ greater than the world price enabling it to exploit its monopsony power with respect to the negative residual demand (that is, positive residual supply) that it faces.

For any equilibrium $\langle X^{*i}(\cdot,\cdot)\rangle_{i=1}^n$ of the specific examples analyzed below we shall see that for each country i, if the realization of its own shock is θ^i, then for every $\bar{p}>0$ there exists a vector of realizations of shocks for the other countries $(\bar{\theta}^1,\ldots,\bar{\theta}^{i-1},\bar{\theta}^{i+1},\ldots,\bar{\theta}^n)$ for which $p^*(\bar{\theta}^1,\ldots,\bar{\theta}^{i-1},\theta^i,\bar{\theta}^{i+1},\ldots,\bar{\theta}^n)=\bar{p}$. From Assumptions 2 and 3 it therefore follows that for each country, every point on the export supply schedule to which it commits (conditional on the realization of its own shock) can arise as an equilibrium outcome. The role of uncertainty is thus to make each country's chosen export supply schedule 'credible', in the sense that every price is potentially a world-market clearing price. And when that price arises, given each country's equilibrium beliefs about how the other countries' quantities of exports will respond to movements in the world-price, it is indeed in each country's best interests to export the quantity corresponding to the export-supply schedule to which it has committed.

3. BILATERAL MONOPOLY

In the bilateral monopoly case, that is, where $n=2$, equation 3 reduces to

$$(4)\qquad \frac{p^*(\theta)-q^{*i}(\theta)}{p^*(\theta)} = -\frac{X^{*j}(p,\theta^j)/p(\theta)}{\partial X^{*j}(p,\theta^j)/\partial p}, \text{ for } i,j=1,2\ i\neq j, \text{ for all } \theta\in\Theta^1\times\Theta^2$$

3.1. *Semi-log Linear Schedules and Specific Trade Taxes.* Consider a particular bilateral monopoly situation where country 1 is always the exporter and country 2 the importer, and where, without ambiguity, the export supply function of 1 is denoted by $X(\cdot,\cdot)$ and the import demand function of 2 is denoted by $M(\cdot,\cdot)$. Assume that

$$(5)\qquad X(p,\theta^1)=\theta^1\exp(ap), a>0 \quad M(p,\theta^2)=\theta^2\exp(-bp), b>0$$

It is straightforward to calculate that a per unit rise in price leads to an a percent rise in the amount of the good supplied competitively by the exporting country and a

b percent decrease in the amount of the good demanded competitively by the importing country.[7]

The unique equilibrium involves using specific trade taxes by the two countries.

RESULT 3.1. *Given the hypotheses of Section* 2 *and the specification of the export and import functions in* 5, *the unique equilibrium in export supply functions can be implemented by a specific export tax* $t_x = 1/b$ *levied by country* 1, *and a specific tariff* $t_m = 1/a$ *levied by country* 2.

PROOF. By definition $X^*(p, \theta^1) = \theta^1 \exp(a[p - 1/b])$, $M^*(p, \theta^2) = \theta^2 \exp(-b[p + 1/a])$, $(p^*(\theta) - q^{*1}(\theta))/p^*(\theta) = 1/(bp^*(\theta))$ and $(q^{*2}(\theta) - p^*(\theta))/p^*(\theta) = 1/(ap^*(\theta))$. Since $\partial \ln X^*(p,(\theta^1)/\partial p = a$ and $\partial \ln M^*(p, \theta^2)/\partial p = -b$, it readily follows that the first-order conditions in 4 hold, and it is straightforward to check that they characterize a global maximum. For uniqueness, fix θ^1. Notice that log supply and log demand are both linear in price. Moreover, log demand is subject to an additive disturbance ($\log \theta^2$). Hence it follows as an immediate generalization of the KM uniqueness result (Proposition 4 [p. 1261]) that $X^*(p, \theta^1)$ is the only supply schedule that is optimal for each possible realization of θ^2. Similarly, fixing θ^2, by an analogous argument we have $M^*(p, \theta^2)$ is the only demand schedule that is optimal for each possible realization of θ^1. □

Notice that although the equilibrium export supply (respectively, import demand) schedule is a function of θ^1 (respectively, θ^2), the particular nature of the price responsiveness of the underlying competitive export supply and import demand schedules allows us to implement the equilibrium in export supply schedules by a pair of specific tax rates that are *independent* of the particular realizations of θ^1 and θ^2.

To provide some intuition for this result, consider the problem facing the exporting country trying to maximize its welfare given that the importing country has committed itself to a specific tariff of size t_m (which need not be $1/a$). The offered import demand is thus:

$$\hat{M}(p, \hat{\theta}^2) = \hat{\theta}^2 \exp(-b[p + t_m]) \tag{6}$$

The inverse demand that the exporting country faces for a particular realization $\hat{\theta}^2$ of θ^2 can be expressed as:

$$p = \frac{1}{b}(\log \hat{\theta}^2) - \frac{1}{b}(\log Q) - t_m \tag{7}$$

Marginal revenue and marginal cost schedules for particular realizations $\hat{\theta}^1$ of θ^1

[7] Notice that the analysis that follows holds for supply schedules of the form $\theta^1 k^{ap}$, with $k > 1$ and $a > 0$, since it can be reexpressed as $\theta^1 \exp(a[\log k]p)$. Similarly, for import demand schedules $\theta^2 k^{-bp}$, with $k > 1$ and $b > 0$.

and $\hat{\theta}^2$ of θ^2 can in turn be expressed as functions that are *linear* in log quantities:

(8)
$$MR \equiv \frac{\partial(pQ)}{\partial Q} = p - \left(\frac{\partial p}{\partial \log Q}\right)\left(\frac{\partial \log Q}{\partial Q}\right)Q$$
$$= p - \frac{1}{b}$$
$$MC = -\frac{1}{a}\log\hat{\theta}^1 + \frac{1}{a}\log Q$$

That is, in price-log quantity space, the MR schedule is obtained by displacing the inverse demand schedule downwards by $1/b$. Graphing these schedules in price-log quantity space, we see in Figure 3.1 that the profit-maximizing price-log quantity combination for the exporting firm is determined from the intersection of the MR and MC schedules. If we let $p(\hat{\theta}^1, \hat{\theta}^2)$ and $q^1(\hat{\theta}^1, \hat{\theta}^2)$, respectively, denote the market clearing world price and exporting country's domestic price, then reading off Figure 3.1 we see that:

(9)
$$p(\hat{\theta}^1, \hat{\theta}^2) - q^1(\hat{\theta}^1, \hat{\theta}^2) = \frac{1}{b}$$

A change in θ^2 just displaces both the MR and inverse demand schedules by the same amount and so the optimal (inverse) export supply schedule that is traced out is one that lies a distance $1/b$ above the MC schedule. This holds for whatever the realization of θ^1 happens to be. A similar diagrammatic treatment can be employed to illustrate that the optimal trade policy for the importing country given the exporting country is levying a specific export tax is in turn to levy a specific tariff of size $1/a$.

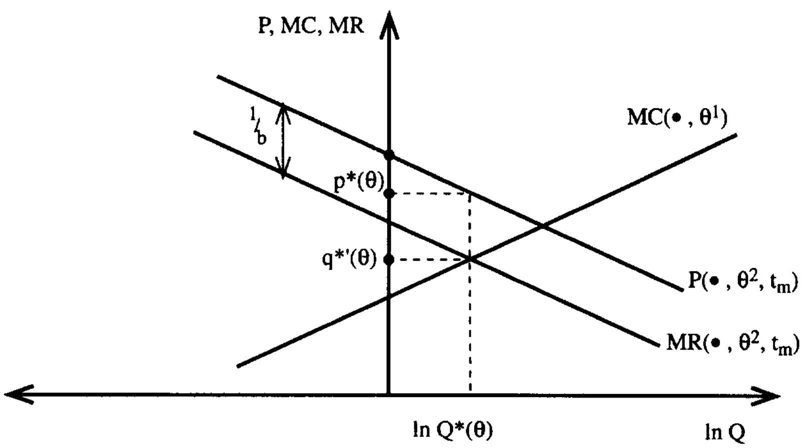

FIGURE 3.1

SEMI-LOG-LINEAR CASE

Of course if we restricted the two countries' governments to select from the set of specific tax rates, the equilibrium of tax rates for the game would be $1/a$ and $1/b$ with or without uncertainty. The significance played by the uncertainty is that this is the *unique* equilibrium pair of trade policies when countries are potentially free to choose any trade policy that can be encoded in a price-quantity schedule. Without uncertainty, any outcome where both countries experience welfare at least as high as their autarkic level could be sustained as an equilibrium in export-supply schedules. Export-supply schedules that involve implausible threats in response to movements in world prices could be sustained in equilibrium, since, without uncertainty, each country is never called upon 'to play' any price-quantity combinations on the equilibrium export-supply schedules to which it has committed, apart from the equilibrium outcome pair itself. By contrast, in an export supply function equilibrium under uncertainty, *every* point on the export-schedule to which a country commits can arise as an outcome for that country in equilibrium.

If we let $Q^*(\theta^1, \theta^2)$ and $p^*(\theta^1, \theta^2)$ denote the equilibrium quantity traded and world price if the two countries adopt their equilibrium strategies, and $Q^c(\theta^1, \theta^2)$ and $p^c(\theta^1, \theta^2)$ the competitive equilibrium quantity traded and world price, it is straightforward to show that the equilibrium in supply functions leads to a constant proportionate decrease in trade relative to the competitive outcome. Moreover, this proportionate decrease is independent of the parameters a or b. To see this notice that the competitive price $p^c(\theta^1, \theta^2) = [\ln \theta^2 - \ln \theta^1]/[a - b]$ while $p^*(\theta^1, \theta^2) = [\ln \theta^2 - \ln \theta^1]/[a + b] + [a - b]/ab$. Hence we have: $Q^*(\theta^1, \theta^2)/Q^c(\theta^1, \theta^2) = 1/e \approx 0{\cdot}37$.

The reduction in the quantity traded obviously entails a lower aggregate welfare associated with the equilibrium in supply schedules relative to the competitive equilibrium. The size of the aggregate welfare loss is greater the less the price responsiveness of the export supply schedule and the import demand schedule, that is, the smaller is a and b. Similarly the side of the market which is relatively more price-responsive is the side for which the terms of trade movement favors in going from the competitive outcome to the outcome for the equilibrium in supply schedules, since $p^*(\theta^1, \theta^2) - p^c(\theta^1, \theta^2) = 1/b - 1/a$.

Although aggregate welfare is unambiguously lower, it is possible that one of the countries might gain in the equilibrium in export supply schedules relative to the competitive equilibrium. To see this, notice that the net welfare change for each country is the difference between the trade tax revenue raised less the loss of net "producer or consumer" surplus compared with the competitive equilibrium. The net welfare change for the exporting country can be explicitly calculated as

$$\frac{1}{b}Q^*(\theta^1, \theta^2) - \int_{q^{*1}}^{p^c} \theta^1 \exp(ap)\, dp = \left[\frac{1}{b} - \frac{1}{a}\left(1 - \frac{1}{e}\right)\right] Q^c(\theta^1, \theta^2)$$

Hence, the exporting country's welfare is higher in the equilibrium in export supply schedules than in the competitive equilibrium if and only if $a/b > e - 1$. Similarly, we can show that the importing country's welfare is higher in the equilibrium in export supply schedules than in the competitive equilibrium if and only if $b/a > e - 1$. Correspondingly, if a/b is in $[1/(e-1), e-1]$ then both countries would prefer to precommit to free trade. In particular, if both the exporting and importing countries exhibit approximately the same absolute price-responsiveness (that is, $a \approx b$) then both prefer to precommit to free trade.

3.2. *Constant Elasticity Schedules and Ad Valorem Trade Taxes.* Consider now a bilateral monopoly situation where both countries competitive schedules' price elasticities are constant. That is, let us assume that

$$(10) \qquad X(p, \theta^1) = \theta^1 p^{\gamma}, \ \gamma > 0 \quad M(p, \theta^2) = \theta^2 p^{-\phi}, \ \phi > 1$$

It is straightforward to calculate that the price elasticity of X is constant and equal to γ while that of M is also constant and equal to ϕ. For this specification, there is also a unique equilibrium in export supply (and import demand) functions that consists of both countries levying ad valorem trade taxes rather than specific taxes as in the previous subsection.

RESULT 3.2. *Given the hypotheses of Section* 2 *and the specification of the export and import functions in* 10, *the unique equilibrium in export supply functions can be characterized by an ad valorem export tax* $\tau_x = 1/(\phi - 1)$ *levied by country* 1 *and an ad valorem tariff* $\tau_m = 1/\gamma$ *levied by country* 2.

PROOF. By definition $X^*(p, \theta^1) = \theta^1[(\phi - 1)p/\phi]^{\gamma}$, $M^*(p, \theta^2) = \theta^2[(\gamma + 1)p/\gamma]^{-\phi}$, $(p^*(\theta) - q^{*1}(\theta))/p^*(\theta) = 1/\phi$ and $(q^{*2}(\theta) - p^*(\theta))/p^*(\theta) = 1/\gamma$. Hence, the first-order conditions of 4 hold, and it is straightforward to check that they characterize a global maximum. For uniqueness, fix θ^1, then, as supply is log-linear and demand is log-linear with an additive disturbance, it follows from a natural generalization of the KM's (1989, Proposition 4, p. 1261) uniqueness result that the only supply schedule that is optimal for each possible realization of θ^2 is $X^*(p, \theta^1)$. Similarly, fixing θ^2, an analogous argument gives $M^*(p, \theta^2)$ as the only demand schedule that is optimal for each possible realization of θ^1. □

In order to provide intuition for this result, consider, as we did in the previous subsection, the problem facing the exporting country trying to maximize its welfare, given that the importing country has now committed itself to an ad valorem tariff of size τ_m (which need not be $1/\gamma$). The offered import demand is thus:

$$(11) \qquad \hat{M}(p, \hat{\theta}^2) = \hat{\theta}^2[(1 + \tau_m)p]^{-\phi}$$

The inverse demand that the exporting country faces for a particular realization $\hat{\theta}^2$ of θ^2 can be expressed as:

$$(12) \qquad p = \left(\frac{1}{1 + \tau_m}\right)(\theta^2)^{1/\phi} Q^{-1/\phi}$$

Marginal revenue and marginal cost schedules for particular realizations $\hat{\theta}^1$ of θ^1 and $\hat{\theta}^2$ of θ^2 can in turn be expressed as:

$$(13) \qquad MR = p(\phi - 1)/\phi \text{ and } MC = (Q/\theta^1)^{1/\gamma}$$

Or in log-linear terms equations (12) and (13) become:

$$(14)\qquad \log p = \log\left(\frac{1}{1+\tau_m}\right) + \frac{1}{\phi}\log\hat{\theta}^2 - \frac{1}{\phi}\log Q$$

$$\log MR = \log p - \log\left(\frac{\phi}{\phi-1}\right)$$

$$\log MC = -\frac{1}{\gamma}\log\hat{\theta}^2 + \frac{1}{\gamma}\log Q$$

That is, plotting these three schedules in log price-log quantity space, we see that all three are linear and that the log MR schedule is obtained by displacing the log inverse demand schedule downwards by $\log(\phi/[\phi-1])$. We see in Figure 3.2 that the profit-maximizing-log price-log quantity combination for the exporting firm is determined from the intersection of the log MR and log MC schedules. If we let $p(\hat{\theta}^1, \hat{\theta}^2)$ and $q^1(\hat{\theta}^1, \hat{\theta}^2)$, respectively, denote the market clearing world price and exporting country's domestic price then reading off Figure 3.2 we see that:

$$(15)\qquad \log p(\hat{\theta}^1, \hat{\theta}^2) - \log q^1(\hat{\theta}^1, \hat{\theta}^2) = \log\left(\frac{\phi}{\phi-1}\right)$$

A change in θ^2 just displaces both the log MR and log inverse demand schedules vertically by the same amount, and so the optimal (inverse) export supply schedule

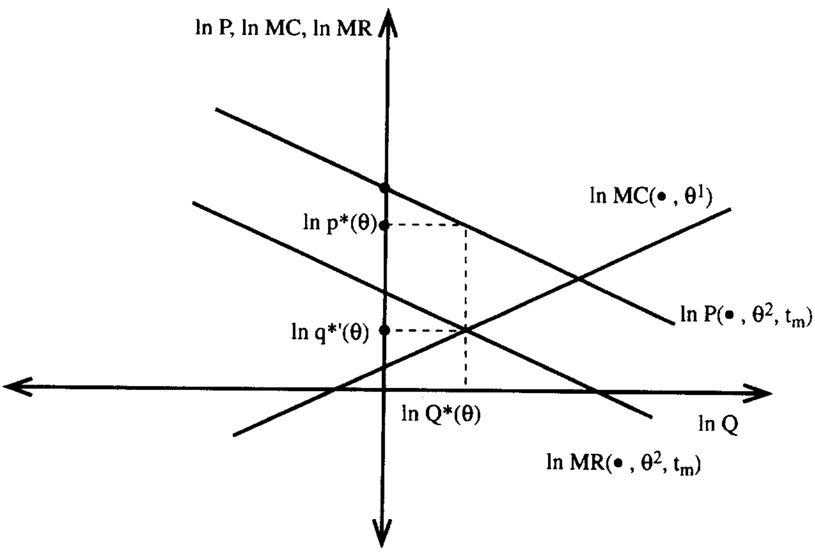

FIGURE 3.2

LOG-LINEAR CASE

that is traced out is one that lies a distance $\log(\phi/[\phi-1])$ above the log MC schedule. That is, the exporting country levies an ad valorem export tax of $1/(\phi-1)$. This holds for whatever the realization of θ^1 happens to be. A similar diagrammatic treatment can be employed to illustrate that the optimal trade policy for the importing country, given that the exporting country is levying an ad valorem export tax, is in turn to levy an ad valorem tariff of size $1/\gamma$.

The total effective tariff rate in this bilateral monopoly equilibrium (that is, $(q^{*2}-q^{*1})/q^{*1}$) is $(\gamma+\phi)/(\gamma\phi-\gamma)$. If we let $Q^*(\theta^1,\theta^2)$ and $p^*(\theta^1,\theta^2)$ denote the equilibrium quantity traded and world price if the two countries adopt their equilibrium strategies, and $Q^c(\theta^1,\theta^2)$ and $p^c(\theta^1,\theta^2)$ the competitive equilibrium quantity traded and world price, it is straightforward to show that the equilibrium in supply functions leads to a constant proportionate decrease in trade relative to the competitive outcome.

$$\frac{Q^*(\theta^1,\theta^2)}{Q^c(\theta^1,\theta^2)}=\left[\left(\frac{\phi-1}{\phi}\right)\left(\frac{\gamma}{\gamma+1}\right)\right]^{\gamma\phi/(\gamma+\phi)} \tag{16}$$

Again, aggregate welfare is obviously less in the equilibrium in export supply functions compared with the competitive equilibrium and, moreover, we can deduce from equation 16 that the relative loss is greater the smaller is ϕ, the price elasticity of the import demand and the smaller is γ, the price elasticity of the export supply.

Although aggregate welfare is unambiguously lower, it is possible that one of the countries might gain in the equilibrium in export supply schedules relative to the competitive equilibrium. As we saw above in the previous subsection, a necessary condition for the trade tax revenue to outweigh the loss of net "producer or consumer" surplus compared with that experienced in the competitive equilibrium, is for the world price (that is, terms of trade) to move in that country's favor. For the constant price elasticity case, the ratio of the world prices can be expressed as

$$\frac{p^*(\theta^1,\theta^2)}{p^c(\theta^1,\theta^2)}=\left[\left(\frac{\phi}{\phi-1}\right)^{\gamma}\left(\frac{\gamma}{\gamma+1}\right)^{\phi}\right]^{1/(\gamma+\phi)} \tag{17}$$

Hence, the terms of trade move in favor of the exporting (respectively, the importing) country the larger (respectively, the smaller) is γ/ϕ. Intuitively, the relatively more price elastic the exporter's supply (respectively, the importer's demand) is for the traded good, the less it gains in the competitive case and the less advantageous it is for the country on the other side of the market to tax trade to exploit its market power. For intermediate values of this ratio, the competitive equilibrium outcome Pareto dominates the outcome of the equilibrium in supply schedules.

4. OLIGOPOLISTIC AND OLIGOPSONISITIC MARKETS

The analysis generally becomes significantly more difficult when the number of countries involved in the market is more than two. Closed-form solutions can be obtained for the case where all (net) competitive-export supply are linear. So we shall assume $X^i(p, \theta^i) = [p - \theta^i]/b^i$, for $i = 1, \ldots, n$.[8] The realization of θ and the associated export supply schedules that the n countries commit themselves to will determine which countries will be exporting and which countries will be importing the good in equilibrium.

Again, for this particular specification the equilibrium in supply schedules is unique and is simple to characterize. In fact the unique equilibrium trade policy of each country can be implemented by a tax that is a quadratic function of the level of trade. That is, each country's chosen supply schedule is linear and goes through the point ($p = \theta^i$, $x^i = 0$), and so can be expressed in the form $X^{*i}(p, \theta^i) = [p - \theta^i]/b^{*i}$.

To see this, let $x^i(\theta)$ denote the equilibrium quantity exported by country i if all countries are choosing such export supply schedules. By definition, for all $i = 1, \ldots, n$ we have:

$$x^i(\theta) = -\sum_{j \neq i} X^{*j}(p(\theta), \theta^j),\ q^i(\theta) = b^i x^i(\theta) + \theta^i \text{ and } p(\theta) = b^{*i} x^i(\theta) + \theta^i.$$

Hence equation 3, the first-order condition that characterizes country i's welfare maximizing point on the 'residual' demand that it faces, can be expressed as:

$$\frac{p(\theta) - q^i(\theta)}{x^i(\theta)} = \frac{1}{\Sigma_{j \neq i}\, \partial X^{*j}(p(\theta), \theta^j)/\partial p} = \frac{1}{\Sigma_{j \neq i}(1/b^{*j})}$$

Or if we let $T^i(x, \theta^i)$ denote i's optimal tax revenue as a function of trade given that the other countries are choosing $X^{*j}(\cdot, \theta^j)$ we have:

$$T^i(x, \theta^i) \equiv (p - q^i)x = \frac{x^2}{\Sigma_{j \neq i}(1/b^{*j})}$$

So indeed a linear export supply schedule is an optimal export supply schedule for i to choose, given that all the others are choosing linear export supply schedules and, moreover, we have from the above two equations that the chosen slopes $\langle b^{*i} \rangle_{i=1}^n$ are defined implicitly as follows:

RESULT 4.1. *Given the hypotheses of Section 2 and that for all* $i = 1, \ldots, n$, $X^i(p, \theta^i) = [p - \theta^i]/b^i$, $b^i > 0$, *then the unique equilibrium in export supply schedules* $\langle X^{*i}(p, \theta^i) \rangle_{i=1}^n$, *takes the form* $X^{*i}(p, \theta^i) = [p - \theta^i]/b^{*i}$, $b^{*i} > 0$ *where* $(b^{*1}, \ldots, b^{*n})$ *satisfies for all* $i = 1, \ldots, n$

$$b^{*i} - b^i = \frac{1}{\Sigma_{j \neq i}(1/b^{*j})} \tag{18}$$

[8] Such an export supply schedule can be derived from a representative consumer with utility function quasi-linear in the numeraire good, y, of the form $U^i(y, x; \theta^i) = y + \theta^i x - \frac{1}{2} b^i x^2$ and cost function (expressed in terms of the numeraire good) $c(x; \theta^i) = \theta^i x + \frac{1}{2} b^i x^2$. That is, the parameter θ^i measures both the intensity of consumer demand and $c'(0; \theta^i)$.

PROOF. The discussion above demonstrates the necessity of 18. Sufficiency is clearly satisfied and uniqueness is again an immediate corollary of KM's (1989, Proposition 4, p. 1261) uniqueness result. □

The two following corollaries are immediate consequences of the set of first-order conditions (18).

COROLLARY 4.1.1. *If there are only two countries, then it follows from the logic of first-order condition* 18 *that the unique equilibrium in export supply schedules is* $X^{*1}(p, \theta^1) \equiv X^{*2}(p, \theta^2) \equiv 0$. *That is, both countries choose infinitely inelastic export supply schedules and no trade takes place in equilibrium.*

COROLLARY 4.1.2. *If all countries have the same absolute price responsiveness for exports (that is,* $b^i = b$ *for all* $i = 1, \ldots, n$*), then for all* $i = 1, \ldots, n > 2$

$$b^{*i} = b^* = \frac{n-1}{n-2} b$$

Interestingly, Corollary 4.1.1 shows that for the bilateral monopoly case with linear competitive-export supply schedules, we obtain the same equilibrium outcome that Rodriguez (1974) and Tower (1975) established for the case of two countries setting trade quotas noncooperatively. In this situation, (18) states that country i wants to commit to an inverse supply that is b^{*j} units steeper than its competitive supply schedule. That is, each country wants to commit to a supply schedule that is steeper than the other's. To see the intuition, consider a situation where country i whose own shock is $\bar{\theta}^i$ and whose competitive export supply schedule has slope b^i faces an inverse demand from the country j with price-intercept $\bar{\theta}^j (> \bar{\theta}^i)$ and finite slope $-\bar{b}^j$. If country i commits to an export supply schedule with slope $\bar{b}^j$, then it exports $\bar{x}^i = (\bar{\theta}^j - \bar{\theta}^i)/(2\bar{b}^j)$ units at a world-price of $(\bar{\theta}^j - \bar{\theta}^i)/2$. But given the slope of the demand curve that it faces, its marginal revenue for the last unit exported is only $\bar{\theta}^i$, which is less than $\bar{\theta}^i + b^i(\bar{\theta}^j - \bar{\theta}^i)$, the domestic opportunity cost of exporting that unit. Hence, country i wants to cut back its trade and so finds it optimal to commit to an export supply schedule with a slope greater than $\bar{b}^j$. An analogous argument holds for a country that finds itself importing the good. Thus each country trying to set a steeper inverse supply schedule than the other leads to the no-trade equilibrium outcome.

Notice that for the 'quadratic' tax revenue function, $b^{*i} - b^i$ measures the second derivative of the tax function with respect to the quantity traded. That is, $b^{*i} - b^i$ is the "acceleration" of the tax function with respect to the increases in the quantity traded. If a country's export supply schedule is more price responsive than another's, that is, say $1/b^i > 1/b^j$, it is fairly intuitive that its equilibrium supply schedule will also be more price responsive than the other's. What also readily follows from the set of first-order conditions (18) is that the acceleration rate of its tax function is also greater.

RESULT 4.2. $1/b^i > 1/b^j$ *implies* $1/b^{*i} > 1/b^{*j}$ *and* $b^{*i} - b^i > b^{*j} - b^j$.

PROOF. From (18) we have

$$b^{*i} - b^i = \frac{1}{\Sigma_{k \neq i,j}(1/b^{*k}) + 1/b^{*j}}$$

$$b^{*j} - b^j = \frac{1}{\Sigma_{k \neq i,j}(1/b^{*k}) + 1/b^{*i}}$$

Suppose $1/b^i > 1/b^j$ and $1/b^{*i} < 1/b^{*j}$, that is $b^j - b^i > 0$ and $b^{*j} - b^{*i} < 0$. But, from the two equations above we have

$$1/b^{*j} > 1/b^{*i} \Leftrightarrow b^{*j} - b^j > b^{*i} - b^i$$

$$\Leftrightarrow b^{*j} - b^{*i} > b^j - b^i$$

A contradiction. □

As a corollary to this result, we note that if two countries experience the same disturbance term, $\theta^i = \theta^j$, then the one with the more price-responsive export supply schedule enjoys the higher level of welfare in the equilibrium in supply schedules. The result states that $1/b^i > 1/b^j$ implies $1/b^{*i} > 1/b^{*j}$. If both countries were exporting in equilibrium, then if country i increased b^{*i} up to b^{*j}, the market clearing price would rise and country j's exports and welfare would rise. As country i would be committed to the same schedule it would also be exporting the same amount as j, but with lower (opportunity) costs would enjoy higher welfare. As b^{*i} and not b^{*j} was chosen by country i, it follows that with the choice b^{*i}, country i's welfare is even greater. A similar argument follows for the situation where both are importing in equilibrium. Of course since $\theta^i = \theta^j$, the two countries are always on the same side of the market regardless of the other countries' disturbances and the supply schedules that they commit to. On the other hand if two country's export supply schedules have the same price responsiveness, then from the geometry of the welfare measures it is straightforward to see that the country whose theta term is further from the equilibrium price is the one with the higher welfare. Summarizing these results we have:

RESULT 4.3. *$b^i \leq b^j$ and $(p^*(\theta) - \theta^i)^2 > (p^*(\theta) - \theta^j)^2$ implies $W^i(\langle X^{*k}(\cdot, \theta^k)\rangle_{k=1}^n, \theta^i) > W^j(\langle X^{*k}(\cdot, \theta^k)\rangle_{k=1}^n, \theta^j)$.*

5. CONCLUDING COMMENTS

Although more complex forms of trade taxes have been known at least since the time of the Corn Laws, governments have generally preferred to levy either specific unit taxes or ad valorem taxes. In this paper, we have presented conditions under which such taxes will be (ex post) optimal for governments seeking to exploit national market power.

In most previous work on strategic trade policy, the form of government intervention has been prescribed in advance. This restriction casts doubt on the generality of the results derived. Particularly noteworthy is the implicit assumption that if one government imposes, say, an ad valorem tariff, the only option available to the other government is to respond in kind, or not at all. The KM solution concept represents a more robust approach to the problem of whether retaliation will eliminate the welfare gain associated with unilateral adoption of strategic trade policies. Although the equilibrium solutions we have derived have involved both parties selecting the same form of intervention, this reflects an endogenous result that stems from the symmetrical assumptions on cost conditions, rather than an exogenous restriction on the set of available strategies.

Given the equilibrium in supply-schedule approach presented in this paper, one could interpret the prevalence around the world of ad valorem trade taxes as evidence that export supply and import demand have constant price-elasticity. However, by reinterpreting the KM equilibrium in export-supply schedules as an equilibrium in *beliefs* among trade-policy makers, it need not be the case that the schedules *literally* exhibit constant price-elasticity, but simply that they are *perceived* by the trade-policy makers as doing so. For example, log-linear specifications are common in econometric estimations of world demand and supply schedules. If the results from such studies were used by the trade-policy makers, those policies would be based on estimated parameters that are constant price-elasticities. In an export-supply schedule setting game, therefore, having (econometrically) estimated the rest of the world's (net) import demand for a country's product as a constant price-elastic schedule (with its exact position subject to a log-linear shock), a country's best response would be to offer an export supply schedule that can be implemented by an ad valorem export tax. The level for this optimal ad valorem export tax would be simply the reciprocal of the estimated price-elasticity of the rest of the world's (net) import demand for that country's product.

Our results, therefore, emphasize our need to understand both demand and supply conditions and, in addition, how the policy makers conceive of these conditions in both their own countries and the rest of the world, before we proceed to game-theoretic modelling of strategic trade theory.

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