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Different notions of disappointment aversion

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Abstract

We discuss three notions of disappointment aversion, due to Gul [1991, Econometrica 59, 667-686], Grant and Kajii [1998, Journal of Economic Behavior and Organization 37, 277-290] and Skiadas [1997, Journal of Economic Theory 76, 242-271; 1997, Econometrica 65, 347-367], explaining how they differ. In the case of Gul and Skiadas we illustrate this difference by means of an example.

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1. Informal discussion

Several recent papers have tried to model the idea of 'disappointment aversion'. Each of these papers is motivated by an informal idea something like the following. When a lottery (or 'act') results in a relatively bad outcome, agents may experience disappointment at not having got a better outcome. This disappointment can worsen the disutility that the outcome produces directly. Similarly, relatively good outcomes can yield pleasurable feelings of 'elation' over and above the utility that the outcomes produce directly. A disappointment-averse agent is one who dislikes disappointment more than she likes elation; this reduces the certainty equivalent value of lotteries or acts.

Disappointment aversion is distinct from risk aversion. Unlike risk aversion, feelings of disappointment (or elation) violate separability axioms that impose that preferences are independent across states: outcomes in events that did not occur affect attitudes towards outcomes that did. Disappoint-

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ment is also distinct from regret. Regret involves comparing outcomes in a given event with those that would have occurred in the same event had the agent chosen a different act or lottery. Disappointment involves comparing outcomes from different events in the same act or lottery. In principle, one could be disappointed without ever having choices to make. Ordinary parlance makes the same distinction between regret and disappointment: Edith Piaf may have had no regrets, but she probably had several disappointments.

The recent papers agree on this underlying idea. The purpose of this note, however, is to illustrate that different formalizations have led to quite different notions of disappointment and of disappointment aversion. In Gul's (1991) and Grant and Kajii's (1998) two notions of disappointment aversion, the differences are clear from inspection. The third notion, due to Skiadas (1997a,b), is formalized in a different framework, so comparison requires more work. This should not cloud the fact, however, that Skiadas' use of the term disappointment aversion is different from the others. To make this clear, we provide examples (in Skiadas' own framework) first of an agent who is disappointment averse in the sense of Skiadas but not in the sense of Gul, and then vice versa.

Recall that, under expected utility, the 'decision weight' on each outcome in a lottery is just the probability of that outcome occurring. One way to model disappointment aversion is to assign larger decision weights to disappointing outcomes and smaller weights to elation outcomes. Both Gul's and Grant and Kajii's models employ this method. The difference lies both in the definition of disappointment and in the weighting schemes.

For Gul, an outcome causes disappointment (elation) if it is worse (better) than the certainty equivalent of the lottery. So for the lottery $P = (x_1, p_1; \dots; x_i, p_i; \dots, x_n, p_n)$ where the probability of getting each outcome x_i is p_i, the contribution that the pair (x_i, p_i) makes to the overall utility $V(P)$ is given by

$$\begin{array}{ll} \left(1 - \dfrac{\mathrm{d}\beta}{1 + \mathrm{d}\beta}\right) p_i u(x_i) & \text{if } u\,(x_i) > V(P) \\ \left(1 + \dfrac{(1 - \mathrm{d})\beta}{1 + \mathrm{d}\beta}\right) p_i u(x_i) & \text{if } u\,(x_i) \le V(P) \end{array}$$

where $u(\cdot)$ is the utility index defined over outcomes, d is the probability that P yields a disappointing outcome (that is, one that is worse than the certainty equivalent of P) and β is the disappointment-averse parameter for the representation. The individual is disappointment averse in Gul's sense if and only if $\beta > 0$. Gul's model is a special case of 'betweenness-satisfying preferences'.[1]

Grant and Kajii do not define disappointment relative to the certainty equivalent. Instead, for them, all outcomes are disappointing relative to the very best outcome in the lottery. In their model the contribution that the pair (x_i, p_i) makes to the overall utility $V(P)$ is given by

$$[(p_i + q_i)^\alpha - q_i^\alpha]u(x_i)$$

where $u(\cdot)$ is again the utility index defined over outcomes, q_i is the probability that the lottery yields

[1]Except for the case of a binary lottery, the probability that the lottery yields a disappointing outcome (i.e. the value of d) is determined endogenously. Hence the representation only implicitly defines the preference functional $V(P)$. But this is typically the case for betweenness-satisfying preferences. For more detail about the structure and properties of betweenness satisfying preferences see Chew (1989) and Dekel (1986).

an outcome worse than x_i, and $\alpha > 0$, is their (relative) disappointment aversion parameter. The individual exhibits (relative) disappointment aversion if and only if $\alpha < 1$. Notice that for fixed (x_i, p_i), its contribution to $V(P)$ is decreasing (increasing) in q_i, if $\alpha < 1$ ($\alpha > 1$). Grant and Kajii's model is a special of 'rank-dependent preferences'.[2]

In Skiadas' framework, an individual is endowed not simply with a single Savage preference relation but with a whole family of 'conditional' preference relations, one for each event. We can think of the preference relation corresponding to the universal event as the unconditional preference relation. The conditional preference relation for an event E can be thought of as the individual foreseeing how he would feel if E occurs. This conditional preference relation can take into account more than just the outcomes on E. For example, it could take into account the disappointment (or elation) in the event E that the outcomes outside of E did not occur.

Suppose acts f and g yield the exact same outcomes on event E, but overall (unconditionally) g is preferred to f. Then, if the individual dislikes disappointment, since all else is equal by construction, she will be less unhappy in the event E if she chose f than if she chose g. That is, conditional on E, she prefers f to g. Skiadas defines such an agent as disappointment averse.

At first glance, this looks very similar to Gul's definition. The similarity seems all the more plausible since, as Skiadas shows (under certain consistency conditions), the unconditional preferences of such a disappointment-averse individual must satisfy betweenness. Recall Gul's definition also implies betweenness.

In fact, the Skiadas notion of disappointment is different. Recall that, for Gul, an outcome is disappointing if it is worse than the certainty equivalent of the given lottery or act. In Skiadas' definition, the outcome of the acts f and g on the event E (the event on which they agree and on which g is relatively disappointing) need not be a bad outcome at all. Indeed, there is nothing in Skiadas' definition to prevent the outcome of g or f on E from being better than any outcomes of g or f outside of E. In this sense, Skiadas' is not a 'within act' notion of disappointment. In Section 2, we formalize Skiadas' definition in his framework, and provide an example to illustrate that a Skiadas disappointment averse individual need not have operational (unconditional) preferences that are Gul disappointment averse.

2. A formal example

Denote by $\mathcal{S} = \{\ldots, s, \ldots\}$ a set of states, by $\mathcal{E} = \{\ldots, A, B, \ldots, E, \ldots\}$ the set of events which is a given σ-field on the universal event $\mathcal{S}$, and by $\mathcal{X} = \{\ldots, x, y, z, \ldots\}$ a set of outcomes (or 'objective' consequences). An act is a (measurable) function $f: \mathcal{S} \to \mathcal{X}$. Let $\mathcal{F} = \{\ldots, f, g, h, \ldots\}$ denote the set of acts on $\mathcal{S}$. We will abuse notation and use x to denote both the outcome x in $\mathcal{X}$ and

[2] Grant and Kajii's can be represented by

$$V(P) = \int_x u(x)\, \mathrm{d}[F_P(x)^{\alpha}]$$

where F_P is the cumulative distribution function associated with the lottery P. For more on the structure and properties of rank dependent preferences, see Quiggin (1993).

the constant act $f(s)=x$ for all s in $\mathscr{S}$. Let $\succeq$ be a binary relation over ordered pairs of acts in $\mathscr{F}$, representing the individual's preferences. Let $\succ$ and $\sim$ correspond to strict preference and indifference, respectively. An event E in $\mathscr{E}$ is said to be *null* (with respect to $\succeq$) if for every pair of acts f and g in $\mathscr{F}$, for which $f(s)=g(s)$ for all s in $\mathscr{S}\backslash E$, we have $f \sim g$. Let $\mathscr{N}$ denote the set of null events.

Recall that the Skiadas decision-maker is endowed with a whole family of 'conditional preferences relations', $\{\succeq^{E}:E\in\mathscr{E}\}$, one for each event. Further recall that the (Savage) unconditional or ex ante preference relation, $\succeq$, is identified with the conditional preference relation, $\succeq^{\mathscr{S}}$, associated with the universal event $\mathscr{S}$.[3]

Skiadas defines the following property.

Strict coherence. A family of conditional preference relations $\{\succeq^{E}:E\in\mathscr{E}\}$ is *strictly coherent* if for any pair of non-null disjoint events A and B in $\mathscr{E}$, and any pair of acts f and g in $\mathscr{F}$:

1. $f\succeq^{A}g$ and $f\succeq^{B}g$ implies $f\succeq^{A\cup B}g$.
2. $f\succ^{A}g$ and $f\succeq^{B}g$ implies $f\succ^{A\cup B}g$.

Skiadas (1997b) formally defines disappointment aversion as follows.

Disappointment aversion. A family of conditional preference relations $\{\succeq^{E}:E\in\mathscr{E}\}$ exhibits *weak* (respectively, *strict*) *disappointment aversion* if for any pair of acts f and g in $\mathscr{F}$, and any event $E\in\mathscr{E}\backslash\mathscr{N}$,

$$(f(s)=g(s) \text{ for every } s \text{ in } E \text{ and } g\succeq f \text{ (resp. } g\succ f)) \text{ implies } f\succeq^{E} g \text{ (resp. } g\succ^{E} g)$$

Consider the following two-parameter class of coherent families of conditional preference relations.

Example. Fix $\mathscr{X}=\mathscr{S}=[0, 1]$. Let μ be the Lebesgue measure on $\mathscr{S}$. Set

$$\varphi_{\beta}(x,w):=\begin{cases}(x-w)(1+\beta) & \text{if } x\leq w\\ (x-w) & \text{if } x>w\end{cases} \text{ where } \beta>-1,$$

and set $v_{\alpha,\beta}(x, w):=\alpha\varphi_{\beta}(x, w)+(1-\alpha)w$ where $\alpha\in[0,1]$. Let $\{\succeq^{E}_{\alpha,\beta}:E\in\mathscr{E}\}$ be the family of preference relations such that, for each E in $\mathscr{E}$, the conditional preference relation $\succeq^{E}_{\alpha,\beta}$ is represented by the functional $V^{E}_{\alpha,\beta}$ defined by $V^{E}_{\alpha,\beta}(f)=\int_{s\in E}v_{\alpha,\beta}(f(s), V_{\beta}(f))\mu$ (ds), where $V_{\beta}(f)$ represents the unconditional preference relation $\succeq_{\beta}$ and solves

$$\int_{\mathscr{S}}\varphi_{\beta}(f(s),V_{\beta}(f))\mu \,\mathrm{d}s=0. \tag{1}$$

Since φ_{β} is decreasing in its second argument, $V_{\beta}(f)$ is well-defined from expression (1) and may

[3]Skiadas does not define outcomes as Savage does, and so in principle his framework is more general than what follows. But this is enough for our purposes.

be viewed as the certainty equivalent outcome for f. That is, for any act f, the constant act which gives the outcome $V_\beta(f)\in(0,\ 1)$ in every s in $\mathcal{S}$, is indifferent to f.

This class of (unconditional) is a special case of Gul's model. They exhibit Gul's notion of disappointment aversion if and only if $\beta>0$. We can view $\varphi_\beta(x,\ y)$ as the 'local utility function' for acts that lie in the same indifference set as the constant act y. If $\beta>0$ then the 'marginal utility' of an outcome in the range of an act from this indifference set is relatively large if that outcome is worse than the certainty equivalent outcome, y, and relatively small if it is a better outcome than y. That is, greater 'weight' is attached to 'disappointing' outcomes in an act than is attached to its 'elation' outcomes.

For any admissible values for α and β, the resulting family of conditional preference relations is, by construction, coherent, since for any pair of disjoint events A and B and any act f, we have $V^{A\cup B}_{\alpha,\beta}(f)=V^{A}_{\alpha,\beta}(f)+V^{B}_{\alpha,\beta}(f)$.

To see under what ranges of the parameter values the coherent family of conditional preference relations is Skiadas-disappointment averse, consider a pair of acts f and g in $\mathcal{F}$, and an event $E\in\mathcal{E}\backslash\mathcal{N}$ such that $f(s)=g(s)$ for every s in E, and $g\succ f$. Let x_f and x_g, denote the certainty equivalent outcomes for f and g, respectively. So, we have

$$
\begin{aligned}
&x_f<x_g \text{ and } V^{E}_{\alpha,\beta}(f)-V^{E}_{\alpha,\beta}(g)\\
&=\int_{s\in E\cap f^{-1}(\{x:x>x_g\})}\left[\alpha\left(f(s)-x_f\right)+(1-\alpha)x_f-\alpha\left(g(s)-x_g\right)-(1-\alpha)x_g\right]\mu\ (\mathrm{d}s)\\
&\quad+\int_{s\in E\cap f^{-1}(\{x:x_g\geq x>x_f\})}\left[\alpha\left(f(s)-x_f\right)+(1-\alpha)x_f-\alpha\left(g(s)-x_g\right)(1+\beta)-(1-\alpha)x_g\right]\mu\ (\mathrm{d}s)\\
&\quad+\int_{s\in E\cap f^{-1}(\{x:x\leq x_f\})}\left[\alpha\left(f(s)-x_f\right)(1+\beta)+(1-\alpha)x_f-\alpha\left(g(s)-x_g\right)(1+\beta)-(1-\alpha)x_g\right]\mu\ (\mathrm{d}s)\\
&=\mu\left(E\cap f^{-1}\left(\{x:x>x_g\}\right)\right)(2\alpha-1)\left(x_g-x_f\right)\\
&\quad+\int_{s\in E\cap f^{-1}(\{x:x_g\geq x>x_f\})}\left[((2+\beta)\alpha-1)\left(x_g-g(s)\right)+(2\alpha-1)\left(g(s)-x_f\right)\right]\mu\ (\mathrm{d}s)\\
&\quad+\mu\left(E\cap f^{-1}\left(\{x:x\leq x_f\}\right)\right)((2+\beta)\alpha-1)\left(x_g-x_f\right)
\end{aligned}
\tag{2}
$$

If the ex ante preference is disappointment averse in the sense of Gul (that is, $\beta>0$) then $\alpha>1/2$ makes all three terms in (2) non-negative with at least one strictly positive, so the family of conditional preferences are strictly disappointment averse in the sense of Skiadas. But suppose that $\alpha<1/2$, and consider an event E satisfying the conditions above and such that $g(s)=f(s)>x_g$ for every s in E. In this case, only the first term in (2) is non-zero, and it is negative: that is $V^{E}_{\alpha,\beta}(f)>V^{E}_{\alpha,\beta}(g)$. Thus, if $\alpha<1/2$, no matter whether or not the ex ante preference relation is Gul-disappointment-averse (that is, regardless of β), the family of conditional preferences is not Skiadas-disappointment-averse. Conversely, if $\alpha=1$ then, no matter whether or not the ex ante preference relation is Gul-disappointment-averse (that is, regardless of β), the family of conditional preferences is Skiadas-disappointment-averse.

Table 1

		Unconditional preference relation	
		Strictly Gul-disappointment-averse	Not strictly Gul-disappointment-averse
Family of conditional preference relations	Strictly Skiadas-disappointment-averse	$\beta > 0$ $\alpha > 1/(2+\beta)$	$\beta \leq 0$ $\alpha > 1/(2+\beta)$
	Not strictly Skiadas-disappointment-averse	$\beta > 0$ $\alpha \leq 1/(2+\beta)$	$\beta \leq 0$ $\alpha \leq 1/(2+\beta)$

Table 1 gives parameter values for α and β for all four possible combinations of Gul and Skiadas disappointment aversions, demonstrating that the notions are formally different.

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