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Corrigendum

Corrigendum to "Bargaining and boldness" [Games Econ. Behav. 38 (2002) 28–51] ☆,☆☆

Albert Burgos,[a] Simon Grant,[b] and Atsushi Kajii [c,*]

[a] *Departamento de Fundamentos del Análisis Económico, Facultad de Economía, Universidad de Murcia, 30100 Espinardo, Spain*
[b] *Department of Economics, Baker Hall, Rice University, Houston, TX 77005, USA*
[c] *Institute of Social and Economic Research, Osaka University, Osaka 567-0047, Japan*

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Abstract

We show the incompatibility between the existence of stationary subgame perfect equilibria in Shaked's game of cycling offers with exogenous breakdown and the behavior of players consistent with the Allais Paradox. Thus, the strategic support of the equally marginally bold solution presented in Burgos, Grant, and Kajii [2002. Games Econ. Behav. 38, 28–51] does not go beyond the two-person case.

JEL classification: C72; C78; D81

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In Burgos et al. (2002) we studied a multi-person bargaining problem with general risk preferences through the use of Shaked's game of cycling offers with exogenous breakdown. Having in mind the class of departures from expected utility shown in the experimental literature, we focussed the game theoretic analysis on preferences accommodating the *certainty effect* or *Allais-Ratio*

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* Corresponding author.
E-mail addresses: albert@um.es (A. Burgos), sgrant@rice.edu (S. Grant), kajii@iser.osaka-u.ac.jp (A. Kajii).

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Paradox. Formally, we assumed (see Burgos et al., 2002, p. 34) that each bargainer i's preferences satisfied.

DICE (Disagreement increasing certainty equivalence). If $\mathbf{y} \sim^i q\mathbf{x}$, then $r\mathbf{y} \preceq^i (rq)\mathbf{x}$ for any probability r.

Under DICE, Lemma 3.1 in Burgos et al. (2002) identified necessary and (supposedly) sufficient conditions for the existence of SSPE offers. After a final version of the paper was sent to the publisher, Oscar Volij pointed out to us that its proof was incorrect. The error can be traced to the fourth stage of the proof (the sufficiency part), in which the constructed strategy does not necessarily constitute a subgame perfect equilibrium if $I \geqslant 3$ and DICE holds strictly. Although we asserted that when $j \neq 1$ is the proposer, the acceptance rule on p. 37 is optimal, it is in fact not necessarily optimal in some subgames which are not along the equilibrium path.

To see why the constructed strategy is not necessarily subgame perfect, assume $I = 3$. Let $\mathbf{x}_i$, $i = 1, 2, 3$, solve the Eqs. (1) on p. 35 of Burgos et al. (2002), and assume every bargainer plays according to the prescribed strategy; that is, bargainer i submits $\mathbf{x}_i$ when she is the proposer and accepts the offer z of bargainer j if and only if $z \geqslant x_j^i$. Pick a small number $\varepsilon > 0$, and consider any subgame starting at the node where player 1 has to respond to player 2's proposal $(x_2^1 - \varepsilon, x_2^2 + \varepsilon, x_2^3)$. According to the constructed strategy, bargainer 1 must reject this offer, and then in the next round bargainer 1 accepts bargainer 3's offer x_3^1. Thus, bargainer 1's continuation payoff at this node is $V^1(\rho, x_3^1)$. On the other hand, if she deviates at this node by accepting, her payoff is $V^1(1, x_2^1 - \varepsilon)$. So we must have $V^1(\rho, x_3^1) \geqslant V^1(1, x_2^1 - \varepsilon)$ for any $\varepsilon > 0$, otherwise bargainer 1 will deviate at this node.

On the other hand, by equations in (1) of Burgos et al. (2002), we have $V^1(1, x_3^1) = V^1(\rho, x_1^1)$ and $V^1(1, x_2^1) = V^1(\rho^2, x_1^1)$. The former equation and DICE imply $V^1(\rho, x_3^1) \leqslant V^1(\rho^2, x_1^1)$. If this holds with strict inequality then by continuity we have $V^1(\rho, x_3^1) < V^1(1, x_2^1 - \varepsilon)$ for small enough ε. This contradicts the inequality $V^1(\rho, x_3^1) \geqslant V^1(1, x_2^1 - \varepsilon)$ assumed above.

Of course if $V^1(\rho, x_3^1) \leqslant V^1(\rho^2, x_1^1)$ holds with the equality, the argument above does not go through. But to guarantee in general $V^j(\rho, x_i^j) = V^j(\rho^2, x_{i+1}^j)$ for any possible ρ and x, we need to postulate preferences such that if $y \sim^i qx$, then $ry \sim^i r(q\mathbf{x})$ for any probability r. This condition implies that preferences are disagreement linear (DL) and thus preferences over elementary lotteries admit a DL-representation of the form $V^i(\rho, x) = \rho v^i(x)$ (see Grant and Kajii, 1995). In this case the model is operationally equivalent to the standard framework with discounted expected utility, since we effectively assume the reduction of compound lotteries axiom (p. 33). Indeed, Volij (2002) points out that if risk preferences are recursively defined, the analysis of alternating offers bargaining is equivalent to the stationary problem studied in Rubinstein (1982) even if reduction is not satisfied.

Since Eqs. (1) in Lemma 3.1 are necessary for the existence of SSPE, it is immediate that when DICE holds with strict inequality (i.e., when preferences display the certainty effect), *there is no SSPE if* $I \geqslant 3$. For the case of two bargainers, however, Lemma 3.1 is correct and, moreover, DICE is not required.

Lemma 3.1 (Corrected). *The proposals* $\mathbf{x}_i$ $(i = 1, 2)$ *corresponding to a SSPE are always accepted, and they are characterized by*

$$V^j\big(1, x_i^j\big) = V^j\big(\rho, x_j^j\big) \quad \textit{for all } i, j. \tag{1}$$

Proof. We shall proceed in four stages. The first three stages go exactly as in Burgos et al. (2002), proving that (i) there is no SSPE where all proposals are rejected, (ii) in a SSPE proposals are always accepted, and (iii) in a SSPE proposals satisfy (1).

Now, it only remains to show that proposals $\mathbf{x}_1$ and $\mathbf{x}_2$ satisfying (1) can be supported in a SSPE. Consider the stationary strategy for bargainer i where, if it is his turn to be the proposer, he proposes $\mathbf{x}_i$, and, if the other individual $j \neq i$, is the proposer, i accepts a proposal $\mathbf{x}_j$ whenever $V^i(1, x_j^i) \geqslant V^i(\rho, x_i^i)$, and rejects otherwise. We shall show that this profile of strategies constitutes an SSPE.

Assume that bargainer 2 follows the stationary strategy above. Consider any subgame which starts with a node where bargainer 1 chooses an action. Suppose there is a strictly preferred strategy for bargainer 1 in that subgame. Since no agreement is the worst outcome, using this strategy bargainer 1 receives $y \in (0, 1]$ in the Tth round from the round to which this node belongs. (That is, $T = 0$ corresponds to the round in which this subgame begins.)

If bargainer 1 is the proposer in that node, then we must have $V^1(\rho^T, y) > V^1(1, x_1^1)$ since he would have received x_1^1 had he followed his putative stationary strategy. It then follows that $y > x_1^1$, and since bargainer 2 always offers x_2^1 and Eq. (1) implies $x_2^1 < x_1^1$, the agreed outcome must be bargainer 1's proposal. But then the share to bargainer 2 is $1 - y < 1 - x_1^1 = x_1^2$; thus bargainer 2 as the responder must have rejected the offer according to the stationary strategy, a contradiction.

So bargainer 1 cannot be the proposer in that node. Let us suppose instead that bargainer 1 is the responder in that node. Denote by $z \in [0, 1]$ the proposal made by bargainer 2 in the preceding node. If $z \geqslant x_2^1$, then bargainer 1's stationary strategy calls for him to accept this offer, and so $V^1(\rho^T, y) > V^1(1, z) \geqslant V^1(1, x_2^1)$. But this implies $y > z$ and $z \geqslant x_2^1$. Therefore $1 - y < 1 - z \leqq 1 - x_2^1 = x_2^2$. This contradicts the fact that bargainer 2, according to his stationary strategy, never accepts any offer below x_2^2 and never offers to bargainer 1 anything different from x_2^1. On the other hand, if $z < x_2^1$, bargainer 1 is to reject it according to the stationary strategy and receives x_1^1 in the next round. So $V^1(\rho^T, y) >$

$V^1(\rho, x_1^1)$ must hold. Since $V^1(1, x_2^1) = V^1(\rho, x_1^1)$ by Eq. (1), and $T \geqslant 0$, it then follows that $y > x_2^1$. Since bargainer 2 always offers x_2^1 in the subsequent rounds, y must be bargainer 1's offer to himself, which in particular implies that $T \geqslant 1$. Thus from $V^1(\rho^T, y) > V^1(\rho, x_1^1)$ we have $y > x_1^1$, which yields $1 - y < x_1^2$. But according to the stationary strategy, bargainer 2 must reject $1 - y$ at any time. This shows that bargainer 1 could not be the responder either. A symmetric argument for the subgame starting with a node where bargainer 2 makes the decision completes the proof. □

In conclusion, the characterization result (Proposition 3.1 in Burgos et al., 2002) for SSPE outcomes is not correct unless $I = 2$. However, since Proposition 3.1 establishes the uniqueness of the solution of (1), the studied bargaining outcome is still the unique outcome supported as a Nash outcome with a stationary strategy profile which is also subgame perfect if preferences are of the expected utility type. If this Nash outcome is denote by $\mathbf{x}(\rho)$, the limit result (Proposition 5.1) goes through even if $I \geqslant 3$. The characterization results on equally marginally bold outcomes (Propositions 4.1 and 4.2) are valid without any modification.

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