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Public Investment and the Risk Premium for Equity

By Simon Grant†‡ and John Quiggin‡

† *Tilburg University* ‡ *Australian National University*

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Analysis of the equity premium puzzle has focused on private-sector capital markets. However, the existence of an anomalous equity premium raises important issues in the evaluation of public-sector investment projects. These issues are explored below. We begin by formalizing the argument that an equity premium may arise from uninsurable systematic risk in labour income, and show that, other things being equal, increases in public ownership of equity will improve welfare, up to the point where the equity premium is eliminated. Finally, we consider policy implications and the optimal extent of public ownership.

Introduction

Since its discovery by Mehra and Prescott (1985), the equity premium puzzle has generated a large literature. The core of this puzzle is the observation that the premium between the rates of return to equity and to debt is much greater than can be explained on the basis of standard models of life-cycle optimization. Many candidate resolutions have been offered. Among the most promising is the suggestion by Mankiw (1986) that capital markets do not spread risk perfectly, and, in particular, that systematic risk is concentrated *ex post* on a small number of people. When this idea is incorporated into a model with heterogeneous individuals (Constantinides and Duffie 1996), the results are consistent with the emergence of an equity premium.

Despite the large literature generated by the equity premium puzzle, remarkably little attention has been paid to the welfare and policy implications of an anomalously large equity premium. One policy issue for which the relevance of the equity premium has been observed is that of social security reform in the United States. A number of writers have argued that, in view of the equity premium, welfare would be increased if individuals were free to invest part or all of their social security accounts in equity. However, as noted by Geanakoplos *et al.* (1998), the diversification of social security investments into stocks has no effect on measures of the 'money's worth' of social security under the assumptions of optimization, time homogeneity, stable prices and spanning. It follows that (assuming that no income redistribution takes place) diversification can have policy-relevant effects only in the presence of some form of market failure.

Grant and Quiggin (2002) explore this point further and evaluate the proposal put forward by the Clinton administration to invest part of the Social Security fund in equities while maintaining the defined-benefit character of the scheme. The effect of this proposal would have been to transfer some of the risk associated with the ownership of equity from individual investors to the public as a whole. Grant and Quiggin show that, if the equity premium arises from market failures such as those considered by Mankiw (1986), public purchases of equity can improve welfare.

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The existence of an anomalous equity premium also raises important issues in the evaluation of public-sector investment projects. Reasoning similar to that of Mehra and Prescott (1985) may be used to support the view that only a small risk premium should be charged for public projects, and therefore that the appropriate rate of discount for public projects is close to the bond rate. On the other hand, in the absence of market imperfections, the discount rate for public projects should be the same as the rate of return on comparably risky private investments.

For representative investments, the private rate of return is well above the bond rate. If the equity premium is the result of imperfections in the private capital market, there is a prima facie case to suggest that the appropriate rate of discount for public-sector investments is that which would be generated by a perfect capital market, rather than the observed rate incorporating the anomalous equity premium. On the other hand, the adoption of a discount rate for public projects that is lower than the rate required by private investors in projects with comparable risk characteristics implies that public projects may displace private alternatives with similar risk characteristics and higher expected returns. Prima facie, this appears undesirable.

These conflicting intuitions are similar to those raised in the debate between Arrow and Lind (1970), who argued that in many cases no risk premium should apply to public investment projects, and writers such as Hirshleifer (1965), who argued that in the absence of market failure public investment projects should be evaluated on the same basis as private projects with similar risk characteristics. This debate was never fully resolved, in large measure because it was not clear what market failures might be relevant.

Arrow and Lind proved that, under standard assumptions, no risk premium should apply for 'small' projects with returns that were uncorrelated with other components of national income and went on to observe: 'Further, if there is some positive correlation between the returns of an investment and other components of national income, the question remains as to whether this correlation is so high as to invalidate the previous result' (1970, p. 373).

Under the standard assumptions of the capital asset pricing model (CAPM), no risk premium should be applicable to a private project if the returns have zero correlation with aggregate income. Hence, in this polar case, there is no inconsistency between the conclusions of Arrow and Lind and those of Hirshleifer. If, however, calculations similar to those of Arrow and Lind are applied to projects where returns have a small positive correlation with other components of national income,[1] using risk preferences similar to those considered by Mehra and Prescott, the Arrow–Lind and Hirshleifer approaches yield different implications. As Arrow and Lind suggest, their result is robust to the existence of the type of modest correlation with national income that characterizes many infrastructure projects, with beta values in the range between 0 and 1. An analysis similar to that of Mehra and Prescott implies that the relevant risk premium should be less than half a percentage point. By contrast, the observed value of the equity premium for the market as a whole, applicable to a project with a beta of 1, is between six and eight percentage points, and the Hirshleifer approach would imply that the same premium should be applied to public projects.

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The debate over the equity premium puzzle suggests the possibility of re-examining the Arrow–Lind proposition in a general equilibrium setting. Supposing that the anomalous equity premium arises from some form of capital market failure, it is necessary to consider whether public investment projects should be evaluated using the market price of risk, so that public and private projects with the same risk and return characteristics should be regarded as perfect substitutes, or whether, as implicitly suggested by Arrow and Lind, the appropriate price of risk for public investment projects is that which would arise in a first-best equilibrium.

These issues are explored in the present paper. We begin by formalizing the argument of Mankiw (1986) in a framework where problems of adverse selection prevent individuals from insuring against systematic risk in labour income, modelled as the return to human capital. It is shown that the absence of risk pooling can generate an equity premium.

Next, we consider the possibility of public ownership of equity and show that, other things being equal, increases in public ownership financed by government borrowing (that is, by selling government bonds) will improve welfare, up to the point where the equity premium is eliminated. The welfare improvement arises from the power the government has to levy taxes in the future to meet any obligation to make specific defined payments in the future, such as those promised to its bondholders.

If the public project displaces a private project with similar state-contingent revenues, then in a boom event revenues are sufficient to pay bondholders and the distribution of the surplus to the private sector leaves everyone as well off as would have been the case if the private-sector project had not been displaced. In a recession event for which the public project's returns are not sufficient to meet payments to bondholders, taxes need to be raised.

If taxes are progressive or proportional, however, the taxes are paid predominantly by the owners and workers of successful firms. This means that, from the *ex ante* perspective of an individual, the spread of the distribution of their after-tax income remains unchanged for a boom event, but has been reduced for a recession event, delivering an increase in their expected welfare provided they are risk-averse.

Finally, we consider policy implications and argue that, in practice, the optimal extent of public ownership will be determined by a trade-off between the advantages of risk spreading associated with public ownership and the superior operating efficiency associated with private ownership in the absence of externalities and other problems requiring regulation.

The central conclusion of the paper is that, in the presence of an inefficiently large risk premium for equity, arising from adverse selection problems, an *ex ante* Pareto improvement may be generated when public projects displace private alternatives with similar risk characteristics and higher expected returns.

I. Model

The critical feature of the model is that individuals cannot fully spread the risk associated with recession, and therefore are less willing to hold equity than they would be in a world of perfect capital markets. As in Mankiw (1986), there are

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two global events: recession, denoted R, and boom, denoted B. The probability of recession is π. Securities can be made contingent on these global events, but not on more finely specified states of nature that affect particular firms and individuals. Like that of Mankiw, the model is atemporal. Thus, we are concerned with the relative prices of bonds and equity, considered as bundles of state-contingent claims, rather than with rates of return over time. This distinction is explored further by Grant and Quiggin (2000), who use the term 'risk premium for equity' to refer to the proportional deviation of the relative price of bonds and equity from that which would apply under risk neutrality. The risk premium for equity is not, in general, equal to the 'equity premium' observed in measures of the rate of return over time. The same terminology will be used in this paper. The implications of extending the analysis to include intertemporal optimization are discussed below.

(*a*) *Individuals and firms*

The production technology and the determination of returns to human and physical capital are not modelled explicitly. The basic idea is that real wages are sticky and that firms hoard labour. The crucial parameters in the model are: h, the share of human capital in total income in the boom event; k, the riskiness of returns to physical (that is, non-human) capital relative to the returns to human capital; π, the probability of recession; and y, the proportion of firms that fail in the recession event.

The model is normalized so that, in the boom event, both aggregate income and the return to human capital are set equal to 1. We assume a continuum of firms F distributed uniformly over the interval $[0, 1]$ and a continuum of individuals I distributed uniformly over the rectangle $[0, \sqrt{2}] \times [0, 1]$. The individual, indexed by (i,j), is assumed to be employed by firm j and endowed with human capital of quality hi. Thus, the total endowment of human capital for the economy is h, consistent with the normalization assumptions. For simplicity of exposition, we assume that each individual i is a risk-averse expected utility maximizer with a concave von Neumann–Morgenstern utility index, u_i. In the derivation of numerical solutions, it will be assumed that preferences display constant relative risk aversion.

There are two types of firm, 'risky' and 'safe'. We assume that firms with an index in the interval $[0, \gamma]$ are risky and that the remainder are safe. We assume further that no one in the economy can determine the identity of a firm before uncertainty is resolved in period 1.

In the boom event, every risky-type firm generates revenue equivalent to $h + (1 - h)k$ units of the single consumption good, where k is greater than one. Every safe-type firm, on the other hand, generates revenue of more modest proportions, equivalent to $h + (1 - h)(1 - k\gamma)/(1 - \gamma)$ units of the consumption good. The wage paid to employee (i,j) by firm j is hi, and the payout by firm j to non-employee claimants is $(1 - h)k$ if $j \leqslant \gamma$, and $(1 - h)(1 - \gamma)/(1 - k\gamma)$ otherwise.

In the recession event risky-type firms go 'bust' and generate no revenue, but safe-type firms still generate the same revenue as in the boom event, paying the same amount to their employees and non-employee claimants. Thus, the nature of the adverse selection problem is similar to that of Stiglitz and Weiss (1981).

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For each i in $[0, 2]$, set $H_i := \{(i', j) : i' = i\}$. That is, H_i is the set of individuals who have human capital of quality hi. We assume that, for each i in $[0, 2]$, every individual (i,j) in H_i has the same preference relation over contingent consumption. Therefore, to avoid the use of the cumbersome index (i,j), we shall identify an individual from the class H_i simply by the index i.

Notice that, since individuals do not know the index of the firm that employs them, from their perspective the return to the human capital of a particular member of the set H_i is random. However, the aggregate (and hence average) return is deterministic and equal to $(1 - \gamma)hi$. Hence the aggregate event payoff vector for all human capital in the economy is $[h, (1 -)h]$.[2] The event-contingent non-wage payment from the economy's aggregate stock of firms is $(1 - h)$ in the boom event and $(1 - h)(1 - k\gamma)$ in the recession event. This implies that the aggregate event payoff vector for the whole economy is $[1, (1 - \delta)]$, where $\delta = \gamma[h + (1 - h)k]$.

If π denotes the probability that period 1 will be in recession, the distribution of aggregate income has parameters: mean $1 - \pi\delta$, standard deviation $\delta\sqrt{[(1 - \pi)\pi]}$ and a coefficient of variation

$$cv = \frac{\delta\sqrt{[(1 - \pi)\pi]}}{1 - \pi\delta}. \tag{1}$$

With $\pi = 0.1$, $h = 0.7$, $\gamma = 0.05$, $k = 5$, δ is approximately 0.117 and cv is approximately 0.033.

(*b*) *The state space*

We derive equilibrium asset prices using the state-claim approach commonly used in finance theory (see e.g. Hirshleifer and Riley 1992). To model the idea of uninsurable systematic risk, it is necessary to specify the structure of the state space in more detail than is common. Since each individual's experience is unique, a full specification of the state space must take the form of a Cartesian product, taken over the set of individuals I, of the features of the world relevant to the individual concerned. Obviously, this set is quite large. Fortunately, the events relevant to asset prices may be specified more simply. Recall that the boom is the same for all individuals, but only a proportion γ of the population is affected by recession. Hence in our model, for any individual i, the state space may be partitioned into the following three events:

(1*i*) boom, occurring with probability $(1 - \pi)$;
(2*i*) recession without job loss, occurring with probability $\pi(1 - \gamma)$;
(3*i*) recession with job loss, occurring with probability $\pi\gamma$.

Job loss in the model arises when firms go 'bust'. Hence for individual i, events (2*i*) and (3*i*) arise from the conjunction of the global event R and the personal event:

(i_0) individual i is employed in a safe firm with probability $(1 - \gamma)$, or
(i_1) individual i is employed in a risky firm with probability γ.

Claims on the income of firms, net of wage payments, are traded in a securities market. There are two types of security: bonds and equity. The bond, which is in

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zero net supply, is risk-free and pays one unit of consumption in period 1 in every state of the world. We think of equity as the sum total of the claims to firms' event-contingent non-wage or non-human capital payments.[3] Thus, assuming that the supply of equity is fixed at one share, equity pays $(1-h)(1-k\gamma)$ in the recession event and $(1-h)$ in the boom event. We further assume that each individual (i,j) is endowed with i units of equity and zero units of bonds.[4]

We assume that trade in securities takes place before individuals have any information about the index of the firm that employs them, and that no trade is possible after this information becomes known. We consider two possibilities in relation to 'personal' contracts, such as insurance contracts, in which the payment to individual i is different in the events i_0 and i_1. In the first case, individuals are free to withdraw from such contracts after receiving private information about the index of their firm. As a result, adverse selection problems ensure that only individuals who find themselves to be employed by a risky-type firm will adhere to insurance contracts (that is, contracts with a positive payout in state $3i$ and a negative payout in the other states). The only sustainable contracts are those with a payout structure that depends only on the occurrence of the global boom and recession states. Such contracts are redundant, since they may be duplicated by an appropriate combination of equity and bonds. Hence no pooling of idiosyncratic risk will take place. In the second case, insurance contracts may be entered into before individuals have information about the index of the firm that employs them, and may not be renegotiated subsequently. As a result, idiosyncratic risk is fully pooled. That is, for each i in $[0,2]$, every individual in the set H_i will 'pool' the returns to their human capital, thereby receiving the (deterministic) return $(1-\gamma)hi$ in the global recession event.[5]

(c) Bonds, equity and security market equilibrium

For any securities market equilibrium there exist (generally non-unique) supporting price vectors $\mathbf{p} \in \mathbb{R}_+^S$ for state-contingent claims, having the property that, for any security k with payoff z_s^k in state s, the price of the security satisfies

$$P^k = \int_S p_s z_s^k \, \mathrm{d}\mu,$$

where μ denotes the probability measure representing likelihoods, defined over the state space. Furthermore, in any securities market equilibrium, the supporting price vector has the property that, for any pair of states s and t in S, and any individual i, there exists a supporting price vector such that

$$\frac{\mu_s u_i'(y_{si})}{\mu_t u_i'(y_{ti})} = \frac{p_s}{p_t},$$

where y_{si} denotes income in state s for individual i. Provided u is strictly concave, the supporting price vector for individual i is unique. Moreover, if the set of securities spans the state-space, there is a unique supporting price vector common to all individuals.

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The same claims are true if the states are replaced by the events $\{1i, 2i, 3i\}$, on which income is constant for any individual. Hence, for any individual i, there exists a unique vector (p_{1i}, p_{2i}, p_{3i}) of event-contingent claim prices with the price ratios equated to the marginal rate of substitution between event-contingent consumption claims. Denote the income vector for individual i by (y_{1i}, y_{2i}, y_{3i}).

If the price of a bond with payoff $(1, 1, 1)$ is normalized to $B = 1$, the supporting event-contingent claims price vector may be derived as the solution of the system:

$$p_{1i} + p_{2i} + p_{3i} = 1 \text{ (by normalization)}$$

$$\frac{p_{2i}}{p_{1i}} = \frac{\pi(1-\gamma)u'(y_{2i})}{(1-\pi)u'(y_{1i})}$$

$$\frac{p_{2i}}{p_{1i}} = \frac{\pi\gamma u'(y_{3i})}{(1-\pi)u'(y_{1i})}.$$

Since the boom event is the same for all individuals, and the set of securities spans the global boom and recession events, in equilibrium there must exist a unique p_l satisfying

$$p_1 = \frac{(1-\pi)u'(y_{1i})}{(1-\pi)u'(y_{1i}) + \pi(1-\gamma)u'(y_{2i}) + \pi\gamma u'(y_{3i})}$$

for all i.[6] This, in turn, implies, from the normalization $B = 1$, that $p_{2i} + p_{3i} = 1 - p_1$ for all i.

Furthermore, with the normalization $B = 1$, the price of a unit of equity may be expressed as

$$\begin{aligned} p_e &= (1-h)p_1 + (1-h)(1-k\gamma)(1-p_1) \\ &= (1-h)[1 - k\gamma(1-p_1)]. \end{aligned} \tag{2}$$

Since the expected return to a bond is 1, and the expected return to a unit of equity is $(1-h)(1-k\gamma\pi)$, the risk premium for equity, expressed in proportional terms, is given by

$$q = \frac{k\gamma[(1-\pi) - p_1]}{1 - k\gamma(1-p_1)}. \tag{3}$$

II. Equilibrium with and without Pooling

In this section we characterize the equilibrium price for a claim on a unit of income in the boom event (and hence for p_e, the price of equity) both for the case in which individuals are free to withdraw from any (mutual) insurance contract after receiving private information about the index of their firm, and for the case in which they cannot withdraw from or renegotiate any such insurance contract. For the reasons discussed above in Section I(b), in the former case no pooling of idiosyncratic risk will take place, while in the latter there is full pooling of idiosyncratic risk among each class H_i of individuals.

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(*a*) *The risk premium for equity without risk pooling*

The portfolio budget constraint in period 0 for individual i is:

$$p_e\theta_i + b_i \leqslant p_e i, \tag{4}$$

where θ_i is the individual's holding of equity and b_i her holding of bonds. Given the portfolio (θ_i, b_i),

$$y_{1i} = hi + \theta_i(1-h) + b_i,$$
$$y_{2i} = hi + \theta_i(1-h)(1-k\gamma) + b_i,$$
$$y_{3i} = \theta_i(1-h)(1-k\gamma) + b_i.$$

As derived above, the price of a claim to consumption in the boom event must satisfy

$$p_1 = \frac{(1-\pi)u_i'[hi + \theta_i(1-h) + b_i]}{E[u'(y)]} \tag{5}$$

for all individuals, where

$$\begin{aligned} E[u'(y)] = {} & (1-\pi)u_i'[hi + \theta_i(1-h) + b_i] \\ & + \pi(1-\gamma)u_i'[hi + \theta_i(l-h)(l-k\gamma) + b_i] \\ & + \pi\gamma u_i'[\theta_i(1-h)(l-k\gamma) + b_i]. \end{aligned}$$

To solve for p_1 (and then for the price of equity, p_e, and the risk premium for equity, q) in the general case requires using (5), (4) and the market-clearing condition for the bond market, $\int_0^1 b_i \,\mathrm{d}i = 0$ (or, equivalently by Walras's law, the market-clearing condition for the equity market, $\int_0^1 \theta_i \,\mathrm{d}i = 1$).

(*b*) *Pooling idiosyncratic human capital risk*

If all individuals were able to pool their idiosyncratic risky returns to their human capital in the recession event, the 'average' return vector to human capital would be $[hi,(1-\gamma)hi]$, yielding the income vector

$$y_{1i} = hi + \theta_i(1-h) + b_i,$$
$$y_{2i} = (1-\gamma)hi + \theta_i(1-h)(1-k\gamma) + b_i,$$
$$y_{3i} = (1-\gamma)hi + \theta_i(1-h)(1-k\gamma) + b_i.$$

Hence the equity price would have to satisfy

$$p_1 = \frac{(1-\pi)u_i'[hi + \theta_i(1-h) + b_i]}{(1-\pi)u_i'[hi + \theta_i(1-h) + b_i] + \pi u_i'[(1-\gamma)hi + \theta_i(1-h)(1-k\gamma) + b_i]}. \tag{6}$$

Mankiw (1986) assumes that u_i' is a strictly convex function for all i, that is, that all individuals exhibit prudence in the sense of Kimball (1990). Under this assumption, for given choices $(\theta_i, b_i,)$, the right-hand side of (5) is less than the

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right-hand side of (6). Hence the absence of the capacity to pool idiosyncratic risk leads to an enhanced risk premium for equity.

By contrast, in the case of linear marginal utility, generated for example by a quadratic utility function, the equilibrium value of p_1 will be unaffected by the presence or absence of pooling. In the absence of pooling, individuals will have a higher willingness to pay for income in the 'job loss' event than in the 'recession without job loss' event, and welfare will therefore be lower than with pooling. Nevertheless, if marginal utility is linear, the average willingness to pay for recession-event income, given by $\pi(1-\gamma)u'(y_{2i}) + \pi\gamma u'(y_{3i})$, will be unaffected by the presence or absence of pooling.

More formally, a pair consisting of a given risk premium for equity, q, and portfolio allocations, $\{(\theta_i, b_i) : i \in [0, 2]\}$, that satisfied (6) and market-clearing for the pooling case, would not constitute an equilibrium for the case where individuals are unable to pool their idiosyncratic human capital risk. The reason is that, given strict convexity of u_i', (5) entails a lower price for a claim for consumption in the boom event (and hence a lower price for equity) than that implied by the risk premium for equity in the pooling case. That is, the expected return on equity that arises in the situation where individuals from each class H_i can pool their human capital risk is insufficient to induce them to hold the quantity θ_i of equity in the case where they cannot pool this risk. Hence with only this premium being offered, the equity market would be in a state of excess supply.

Whether or not individuals exhibited prudence, all individuals would clearly be better off in an *ex ante* sense if they were able to pool their idiosyncratic human capital risk. If, however, knowledge of an individual's human capital parameter is private to that individual, the absence of insurance contracts to provide risk-sharing among individuals could be formally demonstrated by the inability to overcome the adverse selection (or 'lemons') problem that such private information would entail.[7]

(*c*) *Equity premiums for preferences with constant relative risk aversion*

A tractable specific example to consider is the case where every individual exhibits constant relative risk aversion of degree r; that is, for all i, $u_i(y) = y^{l-r}/(1-r)$, $r > 0$, $r \neq 1$, and $u_i(y) = \ln y$, if $r = 1$.

The initial endowments of bonds and equity are proportional to the quality of individuals' human capital. Hence, if preferences are identical and homothetic, the equilibrium allocation involves no trade, whether or not individuals can pool their idiosyncratic human capital risk. That is, for each individual i, (θ_i^*, b_i^*) equals that individual's initial endowment, $(i, 0)$.

Let q^* (respectively, q^+) denote the risk premium for equity that arises in the situation where individuals cannot (respectively, can) pool their idiosyncratic human capital risks.

If $\theta_i^* = i$ and $b_i^* = 0$ we obtain

$$
\begin{aligned}
y_{1i} &= i,\\
y_{2i} &= hi + (1-k\gamma)(1-h)i,\\
y_{3i} &= (1-k\gamma)(1-h)i,
\end{aligned}
$$

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and (5) becomes

$$p_1^* = \frac{(1-\pi)}{(1-\pi)+\pi(1-\gamma)[h+(1-k\gamma)(l-h)]^{-r}+\gamma\pi[(1-k\gamma)(1-h)]^{-r}}.$$

Substituting this expression for p_1^* into (2) and (3) yields p_e^* and q^*, the price of equity and the risk premium for equity, respectively, for the case where individuals *cannot* pool their idiosyncratic human capital risk.

Similarly, in the pooling case:

$$y_{1i} = i,$$

$$y_{2i} = (1-\gamma)hi + (l-k\gamma)(1-h),$$

$$y_{3i} = (1-\gamma)hi + (1-k\gamma)(1-h),$$

and (6) becomes

$$p_1^+ = \frac{(1-\pi)}{(1-\pi)+\pi[(1-\gamma)h+(1-k\gamma)(1-h)]^{-r}}.$$

Substituting this expression for p_1^+ into (2) and (3) yields p_e^* and q^+, the price of equity and the risk premium for equity for the case where individuals *can* pool their idiosyncratic human capital risk.

For the following, not implausible, values of the parameters: $h=0.7$; $\gamma=0.05$; $\pi=0.1$; $k=5$; and a coefficient of relative risk aversion, r, equal to 3, the risk premium for equity when idiosyncratic human capital risk can be pooled is just under 1% ($q^+=q=0.0094$). But without the ability to pool idiosyncratic risks, the risk premium for equity is close to 8% ($q^* = 0.0783$). Given the highly stylized nature of the model presented here, these illustrative values should not be viewed as predictions of the risk premium for equity. Nevertheless, it is encouraging to observe that the addition of idiosyncratic and uninsurable human capital risk can substantially increase the premium on equity over bonds that is required to induce individuals to hold this economy's stock of equity.

III. Public Projects and the Public Sector

We now consider the evaluation of public investment projects. Unlike private investments, the capital required for such projects is assumed to be raised by the issue of public debt, with the government using its power to tax to ensure that the budget balance constraint is satisfied.

We have already observed that agency problems preclude a full pooling of risk through private capital markets and therefore generate a risk premium for equity. Agency problems, such as those arising from the need to monitor firm managers and employees, are also present when governments hold state-contingent claims on the income of enterprises. Although these problems will not be modelled in detail in the present paper, we assume that the existence of agency problems reduces the returns of enterprises relative to the first-best case where no agency problems are present.

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In some cases, for example when the returns to the enterprise depend heavily on regulatory decisions, the agency problems arising from public ownership may be less severe than in the case of private ownership though capital markets. However, for enterprises operating in competitive markets, it is widely believed that the agency problems associated with government ownership are more severe than those associated with private ownership though capital markets. It follows that in such cases the operating efficiency of government enterprises will be less, on average, than that of comparable private enterprises. Studies of the comparative efficiency of public and private enterprises have yielded mixed results. Borcherding *et al.* (1982), who surveyed mainly cases where markets were potentially competitive, reported that most studies found either superior private-sector performance or no significant difference. By contrast, studies of water supply services, a natural monopoly, have generally found either superior public-sector performance (Bhattacharyya *et al.* 1994) or no significant difference (Teeples and Glyer 1987).

Since there is only a single commodity, the outputs of private and public investments in a given event of nature, as measured by the revenues generated by the investments, are perfect substitutes. Moreover, with a fixed stock of human capital and (implicitly) a fixed stock of physical capital, public projects necessarily involve the full crowding out of private projects. We therefore assume that the government uses the proceeds from its bond issue to purchase physical capital which 'crowds out' a proportion of private firms.

In a more general setting, public investment might expand or contract in two different ways. For a given division of the economy into a public sphere (say, infrastructure) and a private sphere (say, manufacturing and services), investment in the public sphere might be increased or reduced with offsetting movements in private investment or consumption. Alternatively, public investment might displace, or be displaced by, private investment in some sphere of activity. The discussion of the Arrow–Lind proposition focused primarily on the first case, reflecting the relatively clear division between public and private-sector roles that prevailed at the time. At present, by contrast, in many sectors of the economy there is a mixture of publicly and privately financed projects producing similar outputs, along with hybrids such as Build, Own, Operate and Transfer (BOOT) schemes.

A full treatment of the topic would require the use of a model with many goods. However, many of the crucial issues can be modelled, using the idea of a menu of public projects. Some investments will yield returns in excess of those in the private sector because, for example, they enable externality problems to be resolved. Others yield lower returns because of agency problems associated with public ownership. We assume that the most attractive investments are undertaken first, and that if a proportion g of private firms are crowded out by the government, the average return to the public sector is $\Phi(g)$ times that in the private sector. The (expected) return on a marginal investment is $\phi(g)$ times that in the private sector, where $\phi(g) = \Phi'(g)$. Thus, a policy in which public projects are undertaken if and only if their returns are greater than or equal to those of private projects with similar risk characteristics would set $\phi(g) = 1$. The Arrow–Lind proposition is that, in the presence of capital market failure, the optimal value of $\phi(g)$ is less than 1.

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(a) *The security market equilibrium with a government sector*

Given a price p_e for equity and 1 for bonds, the market value of the representative private firm, and thus the *ex ante* opportunity cost of not using physical capital in the private sector, is simply p_e. Let b_g denote the government's issue of bonds. The issue of b_g bonds is sufficient to purchase b_g/pe units of equity, which is equivalent to crowding out a proportion b_g/pe of private firms. Hence the financing of a level of public investment equivalent to crowding out a proportion g of the physical capital stock requires (at given asset prices) a government bond issue

$$b_G = p_e g, \tag{7}$$

which may be viewed as the government's portfolio budget constraint.

At the beginning of period 1, given the portfolio it holds from its period 0 gross issue of bonds and purchase of firms, and after it has learned whether the economy is entering a boom or a recession, the government selects a proportional income tax rate to satisfy its *ex post* period 1 budget constraint.

We shall assume that the government-owned firms are a representative sample of all firms in the sense that a proportion γ are risky-type firms and the remainder are safe. They have the same employee profiles as their private-sector counterparts, and we assume that workers receive the same payment in each state of the world that they would receive if the firm were privately run.

In particular, in a boom event, the fraction γ of marginal public investment that has gone into risky firms generates revenue equivalent to $\phi(g)[h + k(1-h)]$ units of the single consumption good, while the remaining proportion $(1-\gamma)$ of marginal public investment that went into safe-type firms generates revenue equivalent to $\phi(g)[h + (1-k\gamma)(1-h)(1-\gamma)]$ units of the consumption good. In the recession event, risky-type firms in the public sector also go 'bust' and generate no revenue (and hence their employees, like their counterparts in risky-type private sector firms, receive no wage payments). But safe-type firms in the public sector still generate the same revenue in the recession event as in the boom event. The aggregate event payoff for the whole economy thus becomes $[\Phi(g) + 1 - g,\ (\Phi(g) + 1 - g)(1-\delta)]$ where (recall) $\Phi(g) = \int_0^g \phi(s)\mathrm{d}s$ and $\delta = \gamma[h + (1-h)k]$.

If we maintain the assumption that all individuals have identical homothetic preferences, then, as was demonstrated in the analysis in Section II(c) above, the demand side of the economy can be represented by a single individual endowed with the economy's aggregate wealth. Hence the representative individual's problem is to choose (b, θ) to maximize

$$(1-\pi)\frac{(c_1)^{1-r}}{1-r} + \pi\frac{(1-\gamma)c_2^{1-r} + \gamma c_3^{1-r}}{1-r}$$

subject to her budget constraint,

$$p_e\theta + b = p_e,$$

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and

$$c_1 = (1 - \tau_B)(h + \theta(1 - h) + b),$$
$$c_2 = (1 - \tau_R)(h + \theta(1 - h)(1 - k\gamma) + b),$$
$$c3 = (1 - \tau_R)(\theta(1 - h)(1 - k\gamma) + b),$$

where τ_B (respectively, τ_R) is the 'tax' rate set by the government in the event of a boom (respectively, recession).

In the boom event, the aggregate pretax income of the private sector is $h + \theta(1 - h) + b$. The government's liability is b_G, its issue of bonds, but it also receives revenues, net of wage payments, from government firms. Hence τ_B is set so that the sum of tax revenue (which is negative in the boom event) and revenues, net of wage payments, from the public firms equals the payment due on the government's issue of bonds:

$$\tau_B[h + \theta(1 - h) + b] + \Phi(g) - gh = b_G. \tag{8}$$

In the recession event, the aggregate pre-tax income of the private sector is $h(1 - \gamma) + \theta(1 - h)(1 - k\gamma) + b$. The government's liability is still b_G, but it receives revenues (net of wage payments) only from the $(1 - \gamma)$ proportion of government-sector firms that do not go bust in the recession. Hence τ_R is set in the recession event so that

$$\tau_R[(1 - \gamma)h + \theta(1 - h)(1 - k\gamma) + b] + \Phi(g)(1 - \delta) - (1 - \gamma)gh = b_G. \tag{9}$$

Given that the government has crowded out a proportion g of private firms through its public investment, in equilibrium $b = b_G$ and $\theta = 1 - g$. Utilizing these market clearing conditions, (8) reduces to

$$(1 - \pi_B)[(1 - g) + gh + b_G] = \Phi(g) + 1 - g. \tag{10}$$

Similarly, (9) simplifies to

$$(1 - \tau_R)[(1 - g)(1 - \delta) + g(1 - \gamma)h + b_G] = (\Phi(g) + 1 - g)(1 - \delta). \tag{11}$$

And the representative individual's event-contingent consumption satisfies

$$c_1 = \Phi(g) + 1 - g,$$
$$c_2 = (\Phi(g) + 1 - g)(1 - \delta) + (1 - \tau_R)\gamma\eta,$$
$$c_3 = (\Phi(g) + 1 - g)(1 - \delta) - (1 - \tau_R)(1 - \gamma)h.$$

(*b*) *The marginal value of government investment*

Suppose (p_1, p_2, p_3) is the event-contingent price vector for the representative individual associated with the security market equilibrium when the government sector is size g. The net value to the representative individual at the margin of government investment (taking into account the opportunity cost of the forgone private investment) can be expressed by

$$MNV(g) = p_1\Delta c_1 + p_2\Delta c_2 + p_3\Delta c_3,$$

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where

$$\Delta c_1 = \phi(g) - 1,$$
$$\Delta c_2 = (\phi(g) - 1)(1 - \delta) - \gamma h \Delta\tau_R,$$
$$\Delta c_3 = (\phi(g) - 1)(1 - \delta) + (1 - \gamma)h\Delta_R.$$

Observation. If $\phi(g) = 1$, then $MNV(g) > 0$.

To see this, notice that

$$\frac{p_2}{p_3} = \frac{(1-\gamma)u'(c_2)}{\gamma u'(c_3)} \qquad \text{and} \qquad c_2 > c_3$$

implies

$$p_2\gamma < p_3(1 - \gamma).$$

The intuition for this result follows from standard risk theory. If there is no efficiency loss associated with public ownership, the expected marginal return to public investment second-order stochastically dominates the expected marginal return to private equity. Hence the welfare of the risk averse representative individual must increase. This, in turn, implies that an expansion of public investment is desirable. For the optimal level of government investment, the marginal investment must have $\phi(g) < 1$; that is, the expected return to marginal public investments must be less than the expected return to marginal private investments.[8]

IV. Alternative Approaches to Risk-Spreading

We have shown that, in the presence of capital market imperfections, the availability of superior risk-spreading through the taxation system may offset differences in technical efficiency between public and private projects. It is natural to ask whether the advantages of risk-spreading can be achieved without incurring the cost of undertaking projects with lower expected returns. In this section we address this question and consider a variety of alternative approaches.

First, consider public policies directed towards reducing the cost of risk-bearing by private firms. The simplest approach would be for governments to offer insurance against losses associated with cyclical fluctuations. However, if it were voluntary, such insurance would face the same adverse selection problems as privately provided insurance. Compulsory public insurance, with the necessary degree of regulation and oversight to mitigate moral hazard problems, would overcome the adverse selection problems.[9] However, the required regulation and oversight would amount to partial or complete public ownership of the project. Hence it is more useful to consider a policy of public purchases of holdings of equity in private enterprises. Such a policy has been proposed for the US social security fund, largely as a response to the observed equity premium, and would yield risk-spreading benefits similar to those arising from investment in public projects (Grant and Quiggin 2002). Public

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holdings of equity in private firms have also arisen as a result of policies of partial privatization in many countries. However, there is no general reason for supposing that the governance costs associated with a given public equity investment would be greater in the case of wholly public projects than if the same investment were dispersed over a number of minority holdings. More generally, the existence of this option can be encompassed in the analysis presented above. It is merely necessary to interpret the class of public investments to include joint ventures and minority holdings in private enterprises.

Next, consider policies directed towards individuals. In principle, it should be possible to design a set of payments to and from individuals that would mimic the risk-spreading benefits of any given public investment project without incurring any efficiency losses. In the single-period two-state model presented here, a tax-financed unemployment benefit paid only in the recession state would have the appropriate properties. Unemployment benefits are broadly equivalent to the application of differential tax rates to individuals facing the personal events $2i$ and $3i$.[10]

However, such policies raise difficulties of their own, particularly when labour supply response is considered. Grant and Quiggin (2002) compare the effects of risk-spreading through public investment policy with an alternative policy, based on the model of Barsky *et al.* (1986), which relies solely on variations in tax rates. Taking labour supply response into account, Grant and Quiggin show that, for plausible parameter values, the public investment policy will yield superior outcomes.

Further issues arise if we consider a multi-period setting in place of the single-period model used here. Assuming that governments pursue a policy of budget balance over the economic cycle, with stable tax rates and rules for welfare benefits, the risk-spreading effects of public investment arise automatically, giving rise to a rule-based rather than a discretionary policy. By contrast, duplication of the risk-spreading effect through the tax–welfare system will require constant fine-tuning of fiscal policy.

Even if it were possible to design a tax-transfer policy that was theoretically equivalent to a given public investment project, the analysis presented in this paper would still be relevant in a second-best sense. The benefit–cost calculation relevant to the evaluation of a given marginal project is based on the distribution of risk that actually exists, as reflected in the observed equity premium, not on the distribution that would arise under a theoretically ideal tax-transfer policy.

V. EXTENSIONS

The static model presented here does not permit consideration of the possibility of smoothing consumption over time through borrowing, lending and the liquidation of assets. Kocherlakota (1996) argues that Mankiw's (1986) solution to the equity premium problem is unsatisfactory because intertemporal consumption-smoothing would overcome any difficulties associated with the absence of insurance markets. Kocherlakota's argument, however, is based on the assumption that individuals can borrow and lend freely at the bond rate. In practice, some individuals are credit-constrained, and all face rates of interest

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for borrowing that are significantly higher than the bond rate, particularly where the purpose of borrowing is to fund current consumption. The same is true for most firms.

On the consumer side, this issue is addressed by Constantinides *et al.* (1998), who use an overlapping-generations model in which credit-constrained young people hold less equity than in the unconstrained optimum. On the producer side, Holmstrom and Tirole (1998) develop an alternative role for liquidity in a model in which firms facing aggregate uncertainty over their liquidity needs are willing to pay a premium for Treasury bonds over private claims.

Since governments can borrow and lend freely at the bond rate, a dynamic analogue to this model, where individuals face idiosyncratic and uninsurable human capital risks that are correlated with the systemic risks to equity that unfold through time, would yield results similar to those derived above. In such a dynamic analogue, the consequence of reducing the government's holding of equity (that is, of 'privatizing') and of outstanding debt would not be neutral. Rather, the price of equity would fall, as would aggregate wealth, signalling the fact that individuals would be worse off by facing a greater exposure to the combination of risks associated with equity and their own human capital.

A second important extension of the model would be the inclusion of agency or moral hazard problems in addition to the adverse selection problem modelled here. Moral hazard problems for individuals could be modelled, as in Grossman and Hart (1983), by assuming that γ, the individual's probability of loss in the global recession event, is not exogenous but depends on unobservable effort.[11] Kahn (1990), using the Grossman–Hart approach, shows that moral hazard problems alone are not sufficient to explain the observed equity premium. However, the interaction between adverse selection and moral hazard problems might produce a richer set of results. It would also be desirable to allow for agency problems on the part of the managers of firms issuing securities. There is, however, no general agreement on the best way of incorporating such problems in a model of security market equilibrium.

The inclusion of agency problems would have important implications for the analysis of government policies based on state-contingent taxes. The incentive effects of taxes on individual effort may be seen as analogous to the agency problems associated with insurance against income losses. Such effects would need to be taken into account in an analysis of the welfare effects of public ownership of equity.

It would also be desirable to take account of agency problems arising from public ownership of equity. The issue of how publicly owned firms might be included in a security market equilibrium model has not, as far as we are aware, been addressed. The incorporation of agency problems, and the question of how the agency problems arising from public ownership of equity differ from those arising from private ownership, would create further difficulties.

VI. Conclusions

In a world of perfect information, risk is of little importance. Although such a world may be characterized by systematic variation in productivity, and therefore in consumption, over a real business cycle, intuition suggests, and the

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analysis of Mehra and Prescott (1985) confirms, that for plausible parameter values the resulting risk premiums are likely to be so small as to be negligible.

The model analysed in this paper contains two deviations from perfect information. The first is the adverse selection problem that prevents pooling of human capital risk. This gives rise to the risk premium for equity. The second is the cost of monitoring and regulating enterprises in the presence of agency problems. In general, agency problems associated with regulation are likely to be smaller for public enterprises, while monitoring problems associated with incentives for effort will be larger for public enterprises. Hence there will exist a range of public projects, of which some will yield higher average returns than the marginal private project and others will yield lower average returns (assuming similar risk characteristics).

The result may be seen as reconciling the views of those, like Arrow and Lind (1970), who argue that the capacity of governments to spread risk through the tax system means that risk premiums for public projects should, in general, be small, and those, like Hirshleifer (1989, p. 111), who argue that, in the absence of capital market imperfections, the risk premium for public and private projects should be the same.

NOTES

1. Or, more precisely, in the terminology of CAPM, a small positive beta.
2. The assumption of a uniform distribution has been made only for convenience in deriving the distribution of aggregate consumption and ensuring that the adverse selection problem prevents the emergence of an insurance market. In what follows, all attention will be on a representative consumer, with mean income equal to aggregate mean income per capita. The use of more realistic distributions, such as the lognormal, would not affect the analysis.
3. The critical distinction between human capital and physical capital is the fact that claims on income from physical capital are traded in the securities market. Thus, in this model the income from unincorporated enterprises would be classed as arising from human capital. Conversely, where claims on labour earnings are tradable, as in the case of rock stars who have 'gone public', the associated income would be classed as arising from physical capital.
4. This assumption incorporates the feature that the endowment of equity and bonds (and hence total wealth) of an individual is positively (and, for simplicity, proportionately) associated with the quality of that individual's human capital. This assumption is designed to allow us to abstract from distributional issues that arise because individuals are differentially endowed with physical and human capital.
5. The restriction on the timing of trade in securities is imposed to permit a simple exposition, in a representative-agent setting, of Mankiw's observation that an equity premium may arise because of a correlation between returns to equity and uninsurable idiosyncratic risk. As discussed below, a more realistic approach would involve a dynamic setting with explicit transactions costs, as in Constantinides and Duffie (1996).
6. Note, by contrast, that different individuals may have different supporting prices for events $2i$ and $3i$.
7. In our model we have also made the endowments of the claims to the two physical assets proportional to the quality of an individual's human capital, thereby making that individual's non-human wealth also proportional to the quality of her human capital. Hence, if non-human wealth were verifiable, there would be no adverse selection problem. But we feel this is relying too literally on the proportional distribution of assets which we assume for the purpose of making analytically tractable the case involving identical homothetic preferences, which is discussed in the next subsection.
8. Since we require only that $\phi(g)$ is non-increasing in g, we cannot infer whether, at the optimal level of government investment, the average expected return on public investment is higher or lower than that on private investments.
9. Such insurance could be mimicked by a tax on equity income, combined with a lump-sum payment to equity owners.

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10. Casual observation suggests that governments tend to be more generous to the unemployed during recessions, so some risk-spreading of this kind is probably already going on, though evidently not enough to eliminate the anomalous equity premium.
11. Alternatively, as in Quiggin and Chambers (1998), unobservable effort could determine state-contingent payoffs.

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