# CEU preferences and dynamic consistency

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## Abstract

This paper investigates the dynamic consistency of Choquet Expected Utility (CEU) preferences. A decision-maker is faced with an information structure represented by a fixed filtration. If beliefs are represented by a convex capacity, we show that a necessary and sufficient condition for dynamic consistency is that beliefs be additive over the final stage in the filtration.
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## 1. Introduction

This paper finds necessary and sufficient conditions for dynamic consistency of Choquet Expected Utility preferences. Schmeidler (1989) proposed Choquet Expected Utility (henceforth CEU) as a theory of choice under ambiguity. However, it also has other applications, for instance, Wu (1999) has used it to model anxiety. Schmeidler's theory did not involve time. To make it more generally applicable, it is desirable to extend it to an

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intertemporal model. Multi-period decisions present new problems. Firstly, individuals will receive information as time progresses. It is necessary to model how they update their beliefs as this information is received. Secondly, it is not clear whether individuals with non-additive beliefs will be dynamically consistent. We consider all updating rules which satisfy a property which we consider to be reasonable.

In the works of Epstein and LeBreton (1993) and Eichberger and Kelsey (1996), it is shown that, under some assumptions, dynamic consistency of CEU preferences implies that beliefs must be additive. However, these papers imposed conditions, which required consistency between different decision trees. In many economic problems, we only need to consider decision-making in a single tree, for instance, any model based on an extensive form game. Hence, it is not clear what implications the earlier results have in this context. Sarin and Wakker (1998) show that for a fixed decision tree, a necessary condition for dynamic consistency is that beliefs be additive except at the final stage. We provide a partial converse to their result by showing that if beliefs are represented by a convex capacity, this condition is also sufficient. In a recent paper, Hanany and Kilbanoff (2004) show, under alternative axioms, how dynamic consistency can be maintained in a fixed decision tree.

It has been argued that non-expected utility preferences are difficult to apply, since they may be dynamically inconsistent, see, for instance, Green (1987) or Hammond (1988). We show that dynamic consistency does not imply beliefs should be additive, however, it does impose some restrictions. How acceptable these restrictions are would depend on the context.

## 2. CEU preferences and dynamic consistency

In this section, we introduce CEU preferences and find conditions for them to be dynamically consistent. We consider a finite set of states of nature $S$. The set of outcomes is a convex set $X \subseteq \mathbb{R}^n$. An act is a function from $S$ to $X$. The set of all acts is denoted by $A(S)$. In this paper, we shall restrict attention to the case where beliefs are represented by convex capacities.

**Definition 2.1.** A convex capacity on $S$ is a real-valued function $v$ on the subsets of $S$ which satisfies the following properties:

1.  $A \subseteq B \Rightarrow v(A) \leqslant v(B)$;
2.  $v(\varnothing)=0$, $v(S)=1$.
3.  $v(A)+v(B) \leqslant v(A \cup B)+v(A \cap B)$, for all $A, B \subseteq S$.

Schmeidler (1989) argues that convex capacities represent ambiguity-aversion. However alternative definitions of ambiguity-aversion due to Epstein (1999) and Ghirardato and Marinacci (2002) have cast doubt on the relationship between convexity and ambiguity-aversion.

If beliefs are represented by a capacity $v$ on $S$, the expected utility of a given act can be found using the Choquet integral.

**Notation 2.1.** Since $S$ is finite, one can order the utility from a given act $a$: $u(a^1) > u(a^2) > \ldots > u\ (a^{r-1}) > u(a^r)$, where $u(a^1), \ldots u(a^r)$ are the possible utility levels yielded by action $a$. Denote by $A^k(a) = \{s \in S | u(a(s)) \geq u(a^k)\}$ the set of states that yield a utility at least as high as $a^k$. By convention, let $A^0(a) = \varnothing$.

**Definition 2.2.** The Choquet expected utility of $u$ with respect to capacity $v$ is:

$$\int u(a(s)) \mathrm{d}v(s) = \sum_{k=1}^{r} u(a^k) [v(A^k(a)) - v(A^{k-1}(a))].$$

Schmeidler (1989), Gilboa (1987) and Sarin and Wakker (1992) provide axioms for representing preferences by a Choquet integral of utility. Another advantage of assuming convexity is that it implies that CEU preferences also have an intuitive multiple priors representation. If beliefs are represented by a convex capacity, $v$ there exists a closed convex set $C$ of probability distributions on $S$, such that: $\int u(a(s)) \mathrm{d}v(s) = \min_{p \in C} \mathbf{E}_p u(a)$.[1] In addition, we shall assume that the utility function is continuous.

**Assumption 2.1.** The utility function $u: X \rightarrow \mathbb{R}$ is continuous.

**Assumption 2.2** (Strong Monotonicity). For two acts $a$, $b \in A(S)$, if $\exists \hat{s} \in S$, such that $u(a(\hat{s})) > u(b(\hat{s}))$ and $\forall s \in S$, $u(a(s)) \geq u(b(s))$ then $a \succ b$.

This says that no state is null in the sense that increasing the utility in any state will lead to a strictly preferred option.[2]

To apply CEU in an intertemporal context, it is necessary to specify how beliefs will be updated as new information is received. There have been a number of proposals for updating CEU preferences, see, for instance, Gilboa and Schmeidler (1993). Instead of focusing on a specific rule, we prove results for any updating procedure which satisfies the following assumption.

**Assumption 2.3.** Let $v$ be a convex capacity on $S$ and let $E$ be an event. Then if $v_\mathrm{E}$ denotes the update of $v$ conditional on $E$, we assume that, $v(E) + v(\neg E) = 1$ implies, $v_E(A) = v(A \cap E) / v(E)$ for $A \subseteq S$.

The strongest motivation for studying Assumption 2.3 is that it is satisfied by the three commonest rules for updating CEU preferences, the Optimistic update, the Dempster–Shafer update, and the Generalised Bayesian Update, (defined below). Thus, using this assumption enables us to prove results for these three rules simultaneously. Assumption 2.3 was motivated by the desire to ensure that the updating rule agrees with Bayesian updating when there is no ambiguity. Since Bayesian updating is agreed to be correct for additive beliefs, it seems reasonable that an updating rule for non-additive beliefs should have this property. If $v(E) + v(\neg E) = 1$, Lemma 2.1 (below) implies that $v(A) = v(A \cap E) + v(A \cap \neg E)$. If $E$ is observed $v(A \cap \neg E)$ is not relevant. Thus,

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[1]This is proved in the Proposition in Schmeidler (1989).
[2]We do not use the full strength of this assumption. In fact we only need it to apply to the events $C$ and $D$ in the proof of Theorem 2.1. There may be some null states, provided these events are non-null.

it does not seem unreasonable to take $v(A \cap E)$ as a measure of the likelihood of $A$. Dividing by $v(E)$ is a normalisation. The Dempster–Shafer update, see Shafer (1976) may be defined as follows.

**Definition 2.3.** Let $v$ be a capacity on $S$. The Dempster–Shafer update (henceforth DS-update) of $v$ conditional on $E \subseteq S$ is given by:

$$v_E(A) = \frac{v((A \cap E) \cup \neg E) - v(\neg E)}{1 - v(\neg E)}.$$

The DS-update has been axiomatised by Gilboa and Schmeidler (1993), where it is shown to be equivalent to a maximum likelihood updating procedure. An alternative is the Optimistic update defined below.

**Definition 2.4.** Let $v$ be a capacity on $S$. If $E$ is observed and $A \subseteq E$, the Optimistic update of $v$ conditional on $E$ is given by: $v_E(A) = \frac{v(A \cap E)}{v(E)}$.

This rule assumes that the worst possible outcome occurred on the complement of $E$, hence the term optimistic. The Generalised Bayesian Update (henceforth GBU) (see Jaffray, 1992; Fagin and Halpern, 1991; Walley, 1991) is defined as follows.

**Definition 2.5.** Let $v$ be a capacity on $S$ and let $E \subseteq S$. If $E$ is observed and $A \subseteq E$, the *GBU* of $v$ conditional on $E$ is given by:

$$v_E(A) = \frac{v(A)}{1 - v(\neg E \cup A) + v(A)}.$$

The GBU can be interpreted as the willingness to pay $p$ for a lottery which pays 1 on $A$ and 0 on $E \backslash A$ and is called off if $\neg E$ occurs:

| | | |
|---|---|---|
| $1-p$ | on | $A$ |
| $-p$ | on | $E-A$ |
| 0 | on | $\neg E$ |

From the CEU of this lottery, $v(A)(1-p)+[1-v(\neg E \cup A)](-p)=0$, one can compute the price $p$ as the likelihood of event $A$ conditional on the event $E$ obtaining.

The following lemma is provides a key step in the proof of the main result.

**Lemma 2.1.** *Let $\mathcal{E}=E_1,..,E_K$ be a partition and let $v$ be a convex capacity on $S$ such that $\sum_{i=1}^{K} v(E_i) = 1$ then for any $B \subseteq S$, $v(B) = \sum_{i=1}^{K} v(B \cap E_i)$.*

**Proof.** First consider the case where $K=2$. Define sets $C$ and $D$ by $C=(B \cap E_1) \cup E_2$, $D=E_1 \cup (B \cap E_2)$. By convexity, $v(C) \geqslant v(B)+v(E_2)-v(B \cap E_2)$, $v(D) \geqslant v(B)+v(E_1)-v(B \cap E_1)$ and $1=v(S) \geqslant (C)+v(D)-v(B)$. Substituting we obtain $1 \geqslant v(B)+v(E_2)-v(B \cap E_2)+ v(B)+v(E_1)-v(B \cap E_1)-v(B)=1+v(B)-v(B \cap E_2)-v(B \cap E_1)$ or $v(B \cap E_2)+v(B \cap E_1) \geqslant v(B)$. However, the opposite inequality follows directly from convexity, which establishes the result in this case: The general result follows by repeated application of the result for $K=2$. $\square$

The following Proposition shows that the DS, optimistic and GBU updates agree with Bayesian updating if a non-ambiguous event is observed.

**Proposition 2.1.** *The DS-rule, the optimistic update and GBU satisfy Assumption 2.3.*

**Proof.** The result is trivial for the optimistic update. Now consider the DS-rule. Let $E$ be an event such that $v(E)+v(\neg E)=1$. For $A \subseteq E$, $v_E(A) = \frac{v(A \cup \neg E)-v(\neg E)}{1-v(\neg E)}$. Let $F=A \cup \neg E$. By Lemma 2.1, $v(F)=v(F \cap E)+v(F \cap \neg E)=v(A)+v(\neg E)$. Hence, $v_E(A) = \frac{v(A)}{v(E)}$.

Now consider the GBU, $v_E(A)=v(A)=v(A)/[1-v(\neg E \cup A)+v(A)]$. By Lemma 2.1, $v(\neg E \cup A)=v(\neg E)+v(A)$. Hence $v_E(A)=\frac{v(A)}{1-v(\neg E)}$. If $v(E)+v(\neg E)=1$ this implies $v_E(A)=\frac{v(A)}{v(E)}$. □

Next we shall find a necessary and sufficient condition for CEU preferences to be dynamically consistent. Let $\mathcal{E}=\langle \mathcal{E}_0,..,\mathcal{E}_T \rangle$ be a filtration. Let $\mathcal{E}_T=\langle E_1^T,...,E_{K_T}^T \rangle$, let $A(E_k^T)$ be the set of acts available after event $E_k^T$ is observed, i.e. $A(E_k^T)$ is a set of functions from $E_k^T$ to $X$. If $E^t \in \mathcal{E}_t$, define $A(E^t) = \times_{E^T \in \mathcal{E}_T, E^T \subseteq E^t} A(E^T)$ to be the set of acts available at time $t$, conditional on event $E^t$ being observed.

**Assumption 2.4.** We assume that the partition $\mathcal{E}_T=\langle E_1^T,...,E_{K_T}^T \rangle$ is non-trivial, i.e. $|E_j^T| \geqslant 2$, for $1 \leqslant j \leqslant K_T$.

**Definition 2.6.** We denote CEU preferences conditional on $E^t \in \mathcal{E}_t$ by $\succeq_{E^t}$. They are defined by: $a \succeq_{E^t} b \Leftrightarrow \int u(a(s)) \mathrm{d}v_{E^t}(s) \geqslant \int u(b(s)) \mathrm{d}v_{E^t}(s)$.

The individual has to choose an act from a set $A(S)$ of acts available at $t=0$. At time $t=\tau(s)$ he receives a signal, that tells him/her in which element of the partition $E \in \mathcal{E}_\tau$ the state of nature lies. Beliefs are then updated and the individual has an opportunity to reconsider his/her decision. If the signal says that the true state of nature is in $E \in \mathcal{E}_\tau$ then (s)he may choose any act from $A(E)$.

The individual formulates a complete contingent plan of action at time $t=0$. After the receipt of new information, (s)he updates his/her beliefs. A new contingent plan is formulated for the remaining time periods. Acts are evaluated by a Choquet integral of utility with respect to the new beliefs. The individual is said to be dynamically consistent if (s)he keeps to his/her original plan. Below we formally define dynamic consistency with respect to a given filtration.

**Definition 2.7.** Preferences are said to be dynamically consistent with respect to a filtration $\mathcal{E}$, if whenever $\tau>t$, $a \succeq_{E^\tau} b$, for all $E^\tau \subseteq E^t$ implies $a \succeq_{E^t} b$.

This definition says that if conditional on any piece of information which might be received, $b$ is not preferred to $a$ then $b$ is initially not preferred to $a$.

The following lemma establishes that, when restricted to non-ambiguous events, any updating-rule satisfying Assumption 2.3 is independent of the order in which information is received.

**Lemma 2.2.** *Let* $\mathcal{E}=\langle \mathcal{E}_0,..., \mathcal{E}_T \rangle$ *be a filtration and let $v$ be a capacity, such that* $\sum_{E \in \mathcal{E}_T} v(E) = 1$. *Let* $E_t \in \mathcal{E}_t$ *and* $E_\tau \in \mathcal{E}_\tau$, *where* $\tau>t$. *Then if Assumption 2.3 is satisfied* $v_{E_\tau^t} = v_{E_t}$.

**Proof.** Consider $A \subseteq E_\tau$. By Assumption 2.3, $v_{E_{t E_\tau}}(A)=\frac{v_{E_t}(A)}{v_{E_t}(E_\tau)}=\frac{\frac{v(A)}{v(E_t)}}{\frac{v(E_\tau)}{v(E_t)}}=\frac{v(A)}{v(E_\tau)}=v_{E_\tau}(A)$. □

Now we present our main result, which establishes a necessary and sufficient condition for CEU preferences to be dynamically consistent. Beliefs must be additive between different members of the finest partition in the filtration. They may however be non-additive within a member of this partition.

**Theorem 2.1.** *Let $\mathcal{E}=\langle \mathcal{E}_0,\ldots,\mathcal{E}_T \rangle$ be a given filtration on S; which satisfies Assumption 2.4. If a decision-maker has CEU preferences with beliefs represented by a convex capacity, which satisfy Assumptions 2.1 and 2.2 and (s)he uses an updating rule which satisfies Assumption 2.3 then the following conditions are equivalent:*

1. *(s)he will be dynamically consistent with respect to $\mathcal{E}$,*
2. $\sum_{E \in \mathcal{E}_T} v(E) = 1.$

**Proof** ($1 \Rightarrow 2$). Suppose that the decision-maker is dynamically consistent. Consider first the case $K=2$. Since the partition is non-trivial, we may find events, $A$, $B$, $C$, and $D$ such that, $E_1=A\cup B$, $E_2=C\cup D$, where $A\cap B=C\cap D=\varnothing$. Consider acts $a$, $b$, $c$, $d$, $e$ and $f$ as described in the following table:

|   | $E_1$ |   | $E_2$ |   |
|---|-------|---|-------|---|
|   | $A$   | $B$ | $C$ | $D$ |
| $a$ | 1 | 1 | 1 | 1 |
| $b$ | 1 | 1 | $\beta$ | 0 |
| $c$ | 0 | 0 | 1 | 1 |
| $d$ | 0 | 0 | $\beta$ | 0 |
| $e$ | $\beta$ | $\beta$ | 1 | 1 |
| $f$ | $\beta$ | $\beta$ | $\beta$ | 0 |

We can ensure that acts with these values exist by appropriately normalising the utility function. Note that $\int a \mathrm{d}v_{E_1}=\int b \mathrm{d}v_{E_1}$, $\int c \mathrm{d}v_{E_1}=\int d \mathrm{d}v_{E_1}$, $\int e \mathrm{d}v_{E_1}=\int f \mathrm{d}v_{E_1}$; $\int a \mathrm{d}v_{E_2}=\int c \mathrm{d}v_{E_2}=\int e \mathrm{d}v_{E_2}$ and $\int b \mathrm{d}v_{E_2}=\int d \mathrm{d}v_{E_2}=\int f \mathrm{d}v_{E_2}$. By continuity and strong monotonicity, we may choose $\beta>1$ so that $\int a \mathrm{d}v_{E_2}>\int b \mathrm{d}v_{E_2}$. Dynamic consistency then implies that $a\sim b$, $c\sim d$ and $e\sim f$. By evaluating the Choquet integrals, we find: $1=(\beta-1)v(C)+v(E_1\cup C)$, $v(E_2)=\beta v(C)$ and $\beta v(E_1\cup C)=\beta v(E_1)+1-v(E_1)$. These equations imply $v(E_1)+v(E_2)=1$.

The general case can be established as follows. If $\sum_{E \in \mathcal{E}_T} v(E)<1$, then we can apply the above argument to $F_1=E_1$ and $F_2 = \cup_{E \in \mathcal{E}_T, E \neq E_1}$ to deduce that dynamic consistency implies $v(F_1)+v(F_2)=1$, which is a contradiction.

$2 \Rightarrow 1$ Now suppose that at time $\hat{t}$ event $\hat{E}$ is observed and at time $\tau > \hat{t}$, $a \succcurlyeq_{\tilde{E}} b$, for all $\tilde{E} \in \mathcal{E}_\tau$, $\tilde{E} \subseteq \hat{E}$. Let $V(a|\tilde{E})=\int u(a)\mathrm{d}v_{\tilde{E}}$ denote the (Choquet) expected utility of $a$ conditional on $\tilde{E}$. By hypothesis and Assumption 2.3, $v_{\hat{E}}(A_i \cap \tilde{E}) = \frac{v_{\hat{E}}(A_i \cap \tilde{E})}{v_{\hat{E}}(\hat{E})}$. Hence

$$v_{\hat{E}}(\tilde{E})V(a|\tilde{E}) = u(x_1)v_{\hat{E}}(A_1 \cap \tilde{E}) + \sum_{i=2}^{m} u(x_i)\left[v_{\hat{E}}(A_i \cap \tilde{E}) - v_{\hat{E}}(A_{i-1} \cap \tilde{E})\right]. \quad (1)$$

Now consider the decision at time $\hat{t}$. By definition, the (Choquet) expected utility of any given act $a$ is equal to $V(a|\hat{E}) = u(x_1)v_{\hat{E}}(A_1) + \sum_{i=2}^{m} u(x_i)\left[v_{\hat{E}}(A_i) - v_{\hat{E}}(A_{i-1})\right]$. Assump-

tion 2.3 implies $v_{\hat{E}}(A)=v(A)/v(E)$, since $v$ is convex, it follows that $v_{\hat{E}}$ is also convex. Lemma 2.1 implies that Eq. (1) may be rewritten as

$$\sum_{\tilde{E}\in\mathcal{E}_{\tau},\tilde{E}\subseteq\hat{E}}\left\{u(x_1)v_{\hat{E}}(A_1\cap\tilde{E})+\sum_{i=2}^{m}u(x_i)\left[v_{\hat{E}}(A_i\cap\tilde{E})-v_{\hat{E}}(A_{i-1}\cap\tilde{E})\right]\right\}.$$

Thus, $V(a|\hat{E})=\sum_{\tilde{E}\in\mathcal{E}_{\tau},\tilde{E}\subseteq\hat{E}}v_{\hat{E}}(\tilde{E})V(a|\tilde{E})$. A similar formula holds for the Choquet integral of $b$. Since for all $\tilde{E}\in\mathcal{E}_{\tau}$, $\tilde{E}\subseteq\hat{E}$, $V(a|\tilde{E})\geqslant V(b|\tilde{E})$, we have $V(a|\hat{E})\geqslant V(b|\hat{E})$ equivalently $a\succcurlyeq_{\tilde{E}}b$, which establishes dynamic consistency. $\square$

**Remark 1.** The strategy of the proof of 1⇒2 is similar to that of Theorem 3.1 in Sarin and Wakker (1998). Some of the assumptions may be relaxed slightly. The proof that 2⇒1 does not make use of the assumptions that utility is strongly monotonic or continuous. The proof that 1⇒2 does not use convexity.

**Remark 2.** From the proof of Theorem 2.1, we can see that Lemma 2.1, which requires beliefs to be represented by a convex capacity, is the most important step. The following example demonstrates that this result is no longer true if we do not assume convexity.

**Example 2.1.** Suppose there are two outcomes Win or Lose, where $u(\text{Win})=1>0=u(\text{Lose})$. Consider a six element state space $S=\{s_1,\ldots,s_6\}$.

The filtration $\mathcal{E}$ on $\langle\{S\}, \{\{s_1, s_3, s_5\}, \{s_2, s_4, s_6\}\}\rangle$. Consider a capacity $v$ on $S$ defined by:

$$v(\{s_1\})=v(\{s_2\})=0.16, \quad v(\{s_i\})=0.15 \text{ for } i\notin\{1,2\}.$$

$$v(\{s_1,s_3\})=v(\{s_1,s_5\})=v(\{s_2,s_4\})=v(\{s_2,s_6\})=0.31,$$

$$v(\{s_3,s_6\})=v(\{s_4,s_5\})=0.34,$$

$$v(\{s_i,s_j\})=0.32, \quad \text{otherwise.}$$

$$v(\{s_1,s_2,s_3\})=v(\{s_2,s_4,s_6\})=0.5,$$

$$v(\{s_i,s_j,s_k\})=0.49 \quad \text{otherwise.}$$

$$v(\{s_i,s_j,s_k,s_l\})=0.68 \text{ for all } i,j,k,l\in\{1,\ldots,6\}.$$

$$v(\{s_i,s_j,s_k,s_l,s_m\})=0.84 \text{ for all } i,j,k,l,m\in\{1,\ldots,6\}.$$

The set of admissible acts are bets on events of the form $\{s_i, s_j\}$, where $(i+j)$ mod 2=1. That is, the individual receives the outcome Win if a state from the event $\{s_i, s_j\}$ obtains, otherwise (s)he receives the outcome Lose.

Clearly, a maximal ex ante strategy is to make a bet on an event $\{s_i, s_j\}$, for which $i+j=9$. Now assume the decision-maker is allowed to make his/her bet on a state in the

element of the partition $\mathcal{E}_1=\{\text{Odd}, \text{Even}\}$, where Odd=$\{s_1, s_3, s_5\}$, Even=$\{s_2, s_4, s_6\}$. By Assumption 2.3, the updated beliefs are given by:

$$v_{\text{Odd}}(\{s_i\}) = \begin{cases} 0.32 & i = 1 \\ 0.30 & i \in \{3, 5\}, \end{cases}, v_{\text{Odd}}(\{s_i, s_j\}) = \begin{cases} 0.62 & \min\{i,j\} = 1 \\ 0.64 & \min\{i,j\} > 1, \end{cases}$$

$$v_{\text{Even}}(\{s_i\}) = \begin{cases} 0.32 & i = 2 \\ 0.30 & i \in \{4, 6\}, \end{cases}, v_{\text{Even}}(\{s_i, s_j\}) = \begin{cases} 0.62 & \min\{i,j\} = 2 \\ 0.64 & \min\{i,j\} > 2. \end{cases}$$

A maximal interim strategy measurable with respect to the partition $\varepsilon_1$ is, bet on $s_1$ if Odd and bet on $s_2$ if Even.

Here is a decision-maker who has CEU preferences with beliefs represented by a capacity which satisfies $v(\{s_1, s_3, s_5\}) + v(\{s_2, s_4, s_6\}) = 0.5 + 0.5 = 1$. The updating rule satisfies Assumption 2.3, yet a maximal ex ante plan must involve a bet on an event $\{s_i, s_j\}$, for which $i+j=9$. However, this is not dynamically consistent, since the strategy which maximizes his updated CEU preferences involves betting on $\{s_1\}$ if $E$=Odd and betting on $\{s_2\}$ if $E$=Even. Theorem 2.1 does not apply since the capacity is not convex,

$$v(\{s_1, s_3, s_5, s_6\}) + v(\{s_2, s_3, s_4, s_6\}) - v(\{s_3, s_6\}) = 0.68 + 0.68 - 0.34 = 1.02 > 1$$

$$= v(S) = v(\{s_1, s_3, s_5, s_6\} \cup \{s_2, s_3, s_4, s_6\}),$$

and so Lemma 2.1 does not hold. To see this, note that $v(\{s_3, s_6\})=0.34$ but $v(\{s_3, s_6\}\cap\text{Odd})+v(\{s_3, s_6\}\cap\text{Even})=v(\{s_3\})+v(\{s_6\})=0.3$.  □

## 3. Conclusion

One of the more common ways to model ambiguity-aversion is to use CEU preferences with a convex capacity. This paper has found conditions under which such preferences will be dynamically consistent. As we have shown dynamic consistency does impose restrictions on CEU preferences. How acceptable these are would depend on the particular application being considered. There are a number of ways in which we could respond to this result.

We could relax dynamic consistency. There is very little experimental evidence which supports the hypothesis that individuals are dynamically consistent. To be convincing, this approach would need to advance strong reasons why individuals might not mind apparent dynamic inconsistencies. Preliminary arguments along these lines can be found in the works of Kelsey and Milne (1999) and Wu (1999).

Another possible reaction is to replace CEU with a different model of ambiguity. The leading contender is the multiple priors model, Gilboa and Schmeidler (1989). As shown by Sarin and Wakker (1998), dynamic consistency imposes a less stringent restriction on the multiple priors model. Pires (2002) has axiomatised an updating rule for such preferences.

If uncertainty is resolved over a period of time, individuals will typically not be indifferent about the time at which uncertainty is resolved. This is related to the issues

discussed in the present paper. Grant et al. (2000) found that additivity over the final partition was also sufficient for CEU preferences to be information-loving. Wu (1999) has shown that a plausible model of preferences concerning the resolution of uncertainty can lead to preferences of the CEU form.

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## References

Eichberger, J., Kelsey, D., 1996. Uncertainty aversion and dynamic consistency. International Economic Review 37, 625–640.

Epstein, L.G., 1999. A definition of uncertainty aversion. Review of Economic Studies 66, 579–606.

Epstein, L.G., LeBreton, M., 1993. Dynamically consistent beliefs must be Bayesian. Journal of Economic Theory 61, 1–22.

Fagin, R., Halpern, J., 1991. A new approach to updating beliefs. Uncertainty in Artificial Intelligence 6, 347–374.

Ghirardato, P., Marinacci, M., 2002. Ambiguity made precise: a comparative foundation. Journal of Economic Theory 102, 251–289.

Gilboa, I., 1987. Expected utility with purely subjective non-additive probabilities. Journal of Mathematical Economics 16, 65–88.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with a non-unique prior. Journal of Mathematical Economics 18, 141–153.

Gilboa, I., Schmeidler, D., 1993. Updating ambiguous beliefs. Journal of Economic Theory 59, 33–49.

Grant, S., Kajii, A., Polak, B., 2000. Temporal resolution of uncertainty and recursive non-expected utility models. Econometrica 68, 425–434.

Green, J.C., 1987. Making book against oneself: the independence axiom and non-linear utility theory. Quarterly Journal of Economics 102, 785–796.

Hammond, P., 1988. Consequentialist foundations for expected utility. Theory and Decision 25, 25–78.

Hanany, E., Kilbanoff, P., 2004. Updating multiple prior preferences, working paper, Northwestern University.

Jaffray, J.Y., 1992. Bayesian updating and belief functions. IEEE Transactions on Systems, Manufacturing and Cybernetics 22, 1144–1152.

Kelsey, D., Milne, F., 1999. Induced preferences, non-additive probabilities and multiple priors. International Economic Review 40, 455–477.

Pires, C.P., 2002. A rule for updating ambiguous beliefs. Theory and Decision 53, 137–152.

Sarin, R., Wakker, P., 1992. A simple axiomatization of non-additive expected utility. Econometrica 60, 1255–1272.

Sarin, R., Wakker, P., 1998. Dynamic choice and non-expected utility. Journal of Risk and Uncertainty 17, 87–120.

Schmeidler, D., 1989. Subjective Probability and Expected Utility without Additivity. Econometrica 57, 571–587.

Shafer, G., 1976. A mathematical theory of evidence. Princeton University Press, New Jersey.

Walley, P., 1991. Statistical reasoning with imprecise probabilities. Chapman and Hall, New York.

Wu, G., 1999. Anxiety and decision making with delayed resolution of uncertainty. Theory and Decision 46, 159–198.