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Contracting under uncertainty: A principal–agent model with ambiguity averse parties

Simon Grant [a,b,*], J. Jude Kline [b], John Quiggin [b]

[a] *Research School of Economics, Australian National University, Acton, ACT 0200, Australia*
[b] *School of Economics, University of Queensland, Brisbane 4072, Australia*

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ABSTRACT

We introduce linguistic ambiguity into a principal–agent contracting framework. Contracts are drafted in a common language. Nevertheless, the principal and the agent may ultimately disagree about the terms of the contract that apply *ex post*. We presume that both parties are ambiguity averse and for tractability reasons that their preferences take a recursive constant absolute risk averse (RCARA) form. We consider various dispute resolution regimes and analyze how the optimal dispute resolution regime depends on the ambiguity attitudes of the parties. We also provide an axiomatization of the class of RCARA preferences.

1. Introduction

The problem of ambiguity in contracts has received a good deal of attention from both economists (beginning with Hogarth and Kunreuther, 1989) and legal scholars (for example Thomas, 2006). Fittingly, however, the term ‘ambiguity’ is itself ambiguous.

In the legal studies literature, as in ordinary language, the term ‘ambiguity’ is used to describe statements which are open to multiple interpretations. The key concern is which interpretation should be preferred in construing the provisions of contracts. We will refer to ‘linguistic ambiguity’ to describe this usage. Grant et al. (2012, 2014) and Halpern and Kets (2015) have developed models of linguistic ambiguity in game theoretic settings. Li (2017) has used empirical methods to elicit ambiguity attitudes when the source of ambiguity is linguistic.

In economics and decision theory, ‘ambiguity’ refers to decision problems in which the probability distribution over states of the world is itself unknown or uncertain. The term is derived from Ellsberg (1961), who uses it to describe “the nature of one’s information concerning the relative likelihood of events ... What is at issue might be called the ambiguity of this information, a quality depending on the amount, type, reliability and ‘unanimity’ of information and giving rise to one’s degree of ‘confidence’ in an estimate of relative likelihoods” (p. 657).

In subsequent decision-theoretic writing, the referent of the term ‘ambiguity’ has shifted, from the information used to derive relative likelihoods to the likelihoods themselves. We will therefore use the term ‘probabilistic ambiguity’ to describe the decision-theoretic usage. A crucial distinction between probabilistic and linguistic ambiguity is that the former is typically interpreted in terms of individual beliefs and preferences, while the latter refers naturally to communication between

* Corresponding author.
E-mail address: simon.grant@anu.edu.au (S. Grant).

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people. In a contracting setting, probabilistic ambiguity is most commonly treated as a property of risks, as perceived by an individual party, while linguistic ambiguity is a property of multiple interpretations of the contract by the parties.

We introduce linguistic ambiguity into a principal–agent contracting framework. Contracts are written in a common language that can express a complete set of mutually and exhaustive contingencies or statements. Each party receives a signal about which of these statements applies. Each party regards his or her own signal as unambiguous, while perceiving the other party's signal to be ambiguous. Although the contractual language is common, the two parties may disagree about its interpretation, that is, which statement applies. This potentially gives rise to contractual disputes. Our focus here is not on the traditional trade-off between incentives and risk, but rather on how different ambiguity and risk attitudes of the parties influence the optimal choice of contracts.

A particularly apt illustration of how linguistic ambiguity can lead to contractual disputes appears in the 2015 Steven Spielberg cold-war thriller "Bridge of Spies". Early in the film, the main protagonist, Jim Donovan, a corporate lawyer (played by Tom Hanks) is seated in a bar engaged in conversation with another lawyer, Bob Bates. They are discussing a claim brought against Jim's client, an insurance company. The company had issued a car-insurance policy to a motorist who subsequently lost control of his car and hit five motorcyclists. The policy limits the company's liability to $100,000 per accident. Bob Bates, representing the five motorcyclists, confidently asserts, "clearly it's five things ... it's self-evident" and so contends the company is liable for up to $500,000. But Jim sees things differently, arguing that "one thing happened, not five things." Thus although both are reading the same contract there is ambiguity about their interpretation of what constitutes "an accident". Bob interprets it as five accidents while Jim sees it as only one. The type of ambiguity described here is what we want to capture in this paper. Notice that it is not probabilistic ambiguity since the two lawyers' disagreement is not about the likelihood of an event but rather the interpretation of which event occurred. Ultimately the consequence that results will depend on how such disputes are resolved. This in turn will affect the type of contracts that are offered and accepted by the parties and the relationship specific investments undertaken by the parties.

We model the agent as having access to a production technology that uses an input to produce a statement contingent output. We consider both statement-contingent contracts and output-contingent contracts. Statement-contingent contracts depend on the signal received. Since the principal and agent may receive different signals that may lead the parties to disagree about which provision of the contract should be operative, these contracts may be subject to disputes. Output-contingent contracts, on the other hand, preclude disputes, since the output is assumed to be observable by both parties.[1]

For statement-contingent contracts we consider various types of dispute resolution. Disputes may be resolved as a 'war-of-attrition' as in Grant et al. (2012, 2014). In this case, the potential loss from ambiguity is maximized and output-contingent contracts may be preferred. Under the standard legal doctrine of *contra proferentem*, however, disputes are resolved against the party who drew up the contract (in this case, the principal).[2] A third alternative, increasingly preferred by those drafting contracts, is to require disputes to be resolved by arbitration panels, which are generally seen as more favorable to principals. We refer to this as mandatory arbitration which we model under the polar assumption that ambiguity is always resolved in favor of the principal.

These are not the only possibilities. One might also envisage contracts that mix the dispute resolution regimes. In particular, taking the legal doctrine of contra proferentem as the default, the contract could specify the event in which the agent has agreed to set aside this doctrine and instead have any dispute resolved by mandatory arbitration. In order to avoid the possibility of a meta-dispute about which dispute resolution regime should operate, we argue that such an event should be *unambiguous* in a sense we define formally in the sequel.

We begin in Section 2 by formally developing the principal–agent framework outlined above. We assume each party's preferences admit a recursive constant absolute risk averse (RCARA) representation. These preferences are particularly tractable since they exhibit three important properties: an own-signal sure thing principle, a conditional sure thing principle and translation invariance. (In Appendix A we show these three properties characterize this class of preferences.) In Section 3 we explore what form contracts might take in this setting. Section 4 provides conditions for various cases in which different dispute resolution regimes are optimally chosen. To help explicate these results, in Section 5 we present an example in which the signal structure has a bivariate normal distribution. Finally, we offer some concluding comments.

2. A model of risk and ambiguity

2.1. The set-up

Two individuals or (potential) parties to a contract, A (an agent) and P (a principal), share a common language for expressing the contingencies or *statements* that can be included in any contract they can draft. We take the most refined set of mutually exclusive and exhaustive statements expressible in this language to correspond to the finite space S. Although they use the same language, the two parties may disagree as to which statement obtains. In particular, when A perceives

[1] In a principal–agent setting, Macleod (2003) considers ambiguity that arises from subjective evaluations of output. While this is distinct from the approach taken here, it also is a potential source of conflict and has implications for the design of optimal contracts.

[2] We thank Daniel Quiggin for bringing this doctrine to our attention. Board and Chung (2009) discuss this doctrine in terms of their object-based model of differential awareness.

statement $s \in S$ to be true, P may perceive $s' \neq s$ to have obtained. We express the pair of signals received by the two parties by a state (of the world) $\omega = (s^{A}, s^{P})$ in the state-space:

$$\Omega = S^{A} \times S^{P} \text{ where } S^{A} = S^{P} = S.$$

Let f denote a random variable defined on Ω. We write $\mathcal{F}$ for the set of all random variables. Let f^{A} (respectively f^{P}) denote a random variable that is measurable with respect to S^{A} (respectively, S^{P}) and let $\mathcal{F}^{A}$ (respectively, $\mathcal{F}^{P}$) denote the corresponding set of such random variables. That is, for any f^{A} in $\mathcal{F}^{A}$ and any f^{P} in $\mathcal{F}^{P}$, we have $f^{A}\left(s, s'\right) = f^{A}\left(s, \hat{s}'\right)$ and $f^{P}\left(s, s'\right) = f^{P}\left(\hat{s}, s'\right)$, for all $s, \hat{s}$ in S^{A} and all $s', \hat{s}'$ in S^{P}. With slight abuse of notation, any consequence $c \in \mathbb{R}$ will also denote the constant random variable that yields c no matter which state $\left(s, s'\right) \in \Omega$ obtains. Let $\mathcal{C}$ denote the set of constant random variables. Notice that $\mathcal{C} = \mathcal{F}^{A} \cap \mathcal{F}^{P}$.

For any two random variables f and f' in $\mathcal{F}$ and any event $E \subseteq \Omega$, let $f_E f'$ denote the random variable defined as:

$$f_E f'(\omega) = \begin{cases} f(\omega) & \text{if } \omega \in E \\ f'(\omega) & \text{if } \omega \notin E. \end{cases}$$

A complete and transitive preference relation $\succsim^{i}$ defined on the set of random variables $\mathcal{F}$ is associated with each party $i =$ A,P. As usual $\succ^{i}$ and $\sim^{i}$ denote its asymmetric and symmetric parts, respectively. An event $E \subseteq \Omega$ is *null* for party i if $f'_E f \sim^{i} f$, for all $f, f' \in \mathcal{F}$. Let $\mathcal{N}^{i}$ denote the set of null events for party $i =$ A,P. For each signal $s \in S$, we define the *conditional possibility event of the agent* $S_s^{P} := \left\{s' \in S : \left\{\left(s, s'\right)\right\} \notin \mathcal{N}^{A}\right\}$ and the *conditional possibility event of the principal* $S_s^{A} := \left\{s' \in S : \left\{\left(s', s\right)\right\} \notin \mathcal{N}^{P}\right\}$. That is, S_s^{P} (respectively S_s^{A}) is the set of realizations of the other signal that the agent (respectively, principal) thinks possible given the realization s of the own signal.

We assume that the each party perceives a “possibility” of agreement at each $s \in S$ in the sense that:

$$s \in S_s^{A} \cap S_s^{P} \tag{1}$$

Notice that this implies that for each statement $s \in S$, the events $\{s\} \times S$ and $S \times \{s\}$ are both non-null. We refer to this property as *every own-signal realization is essential.*

Finally, we require the conditional possibility events of the two parties to be coherent in the sense that for all $s, s' \in S$:

$$s \in S_{s'}^{A} \Leftrightarrow s' \in S_s^{P}. \tag{2}$$

The assumption (2) ensures that the parties agree on the null events, that is, $\mathcal{N}^{A} = \mathcal{N}^{P}$.[3] In the sequel we use $\mathcal{N}$ to denote the set of null events.

2.2. RCARA certainty equivalent representation of preferences

The preferences of the agent and the principal over random variables in $\mathcal{F}$ admit what we dub *a continuous certainty equivalent representation of RCARA preferences*. Specifically, the certainty equivalents are given by:

1. for the agent,

$$e^{A}(f) = -\frac{1}{\alpha_0^{A}} \ln\left(\sum_{s \in S} \mu_{S^{A}}^{A}(s) \left(\sum_{s' \in S_s^{P}} \mu_{S^{P}}^{A}\left(s'|s\right) \exp\left(-\alpha_s^{A} f\left(s, s'\right)\right)\right)^{\frac{\alpha_0^{A}}{\alpha_s^{A}}}\right),$$

   where:

$$\alpha_0^{A} > 0 \text{ and } \alpha_s^{A} \geqslant \alpha_0^{A} > 0 \text{ for every } s \in S^{A}, \tag{3}$$

$$\mu^{A}(\cdot,\cdot) \text{ is a probability distribution on } \Omega \text{ with marginals } \mu_{S^{A}}^{A}(\cdot) \text{ and } \mu_{S^{P}}^{A}(\cdot), \tag{4}$$

$$\text{and conditional distribution } \mu_{S^{P}}^{A}(\cdot|s) \text{ on } S^{P} \text{ with support } S_s^{P}. \tag{5}$$

2. for the principal,

$$e^{P}(f) = -\sum_{s \in S} \mu_{S^{P}}^{P}(s) \frac{1}{\alpha_s^{P}} \ln\left(\sum_{s' \in S_s^{A}} \mu_{S^{A}}^{P}\left(s'|s\right) \exp\left(-\alpha_s^{P} f\left(s', s\right)\right)\right),$$

[3] The restrictions (1) and (2) were explored in Grant et al. (2012) and correspond to reflexivity and complementary symmetry, respectively, of the possibility of dispute relations introduced there.

where:

$$\alpha_s^{\mathrm{P}} \geqslant 0 \text{ for every } s \in S, \tag{6}$$

$$\mu^{\mathrm{P}}(\cdot,\cdot) \text{ is a probability distribution on } \Omega \text{ with marginals } \mu^{\mathrm{P}}_{S^{\mathrm{A}}}(\cdot) \text{ and } \mu^{\mathrm{P}}_{S^{\mathrm{P}}}(\cdot), \tag{7}$$

$$\text{and conditional distribution } \mu^{\mathrm{P}}_{S^{\mathrm{A}}}(\cdot|s) \text{ on } S^{\mathrm{A}} \text{ with support } S^{\mathrm{A}}_s. \tag{8}$$

The parameter restrictions in (3) entail the agent is strictly risk averse (in the usual absolute sense). Furthermore, for each realization of the agent's own signal she is conditionally more averse to the risk associated with random variables that are measurable with respect to the other's signal than she is for ones that are measurable with respect to her own signal. Following Chew and Sagi (2008) and Abdellaoui et al. (2011) we interpret this (conditional) differential degree of risk aversion as a manifestation of her aversion to the ambiguity she perceives there to be with regard to the realization of the principal's signal. Turning to the principal, we see that for any random variable that is measurable with respect to his own signal, its certainty equivalent is simply its expected value. The parameter restrictions in (6) imply, for each realization of his own signal, he is averse to the ambiguity he perceives there to be with regard to the realization of the agent's signal.

Expression (4) specifies the beliefs of the agent and (5) the support of the conditional beliefs of the agent for each statement s. The corresponding beliefs of the principal are described by (7) and (8).

We also assume that the marginals of the parties agree:

$$\mu^{\mathrm{A}}_{S^{\mathrm{A}}}(s) = \mu^{\mathrm{A}}_{S^{\mathrm{P}}}(s) = \mu^{\mathrm{P}}_{S^{\mathrm{A}}}(s) = \mu^{\mathrm{P}}_{S^{\mathrm{P}}}(s) = \mu(s) \text{ for all } s \in S. \tag{9}$$

Recall that the parties agree about the language S for the contract. Assumption (9) requires them to also agree on the likelihood of the occurrence of each statement s in the language. We make this assumption since we want to focus on the effects of differences in ambiguity and risk attitudes on the contract choice, not on differences in the beliefs over the common language.

The certainty equivalent forms we have chosen for the individuals satisfy three important properties: an own-signal sure thing principle, a conditional sure thing principal and translation invariance. We formally define these three properties in Appendix A where we show that within the class of preferences that admit a continuous certainty equivalent representation these three properties characterize the subclass of RCARA preferences.

2.3. Production technology

The agent has access to a production technology that uses an input $x \in X \subseteq \mathbb{R}_+$ to produce a statement contingent output vector $\mathbf{z} \in Z \subseteq \mathbb{R}^{|S|}$. The agent, having committed to produce $\mathbf{z}$, can describe this by saying, 'if statement s is true, I will produce $\mathbf{z}_s$,' while the principal, upon receipt of the signal realization s', expects the agent to produce $\mathbf{z}_{s'}$.

The technology is characterized by the input requirement function $x(\mathbf{z})$, which we take to be equal to the agent's cost of effort. For compactness, we take $Z = [-\overline{z}, \overline{z}]^{|S|}$ for some $\overline{z} > 0$. Following Chambers and Quiggin (2000), we presume that $x(\mathbf{z})$ is non-decreasing and convex.

From the outside observer's viewpoint, one might argue the agent has more precise information about the *ω-state contingent production.* However, from a *preference viewpoint* that is not how the principal views it. And, as we shall see in the sequel, it is the preferences of the two parties that guide the choice of contract. In this respect the RCARA preferences are symmetric in the sense that each party's own information is accorded a more privileged position than the information of the other party.

3. Contracts

We consider two polar forms of contracts in which the principal may choose to make his payment to the agent contingent either on which statement in S obtains or on the output she produces. The advantage of the statement-contingent contract is that the agent's reward need not be made to depend on the output or the statement, thereby allowing the contract to reduce the risk to which the agent is exposed and eschewing the need for any IC constraint to induce the desired statement contingent production plan. However, this comes at the cost of potential disputes arising should the statements the agent and the principal perceive to have obtained not agree.[4]

Who bears the ambiguity depends on how and in whose favor such disputes are resolved. The advantage of the output-contingent contract is that it avoids any possibility of dispute, albeit at the cost of having to impose risk on the agent to induce her to commit the input required to generate the statement-contingent output that the contract is designed to deliver, and of exposing the principal to ambiguity since production and payments are measurable with respect to the agent's signal.

For an agent operating in our setting with (certainty equivalent) outside option $\underline{c}$, we shall consider optimal contracts of each type. We give results on the domination of one contract type over another.

[4] The International Monetary Fund (2017) discusses the practical operation of state-contingent contracts for sovereign debt.

In the absence of wealth effects, the form of the optimal contract does not depend on bargaining power. The quasi-linearity of certainty-equivalent utility with respect to non-state-contingent transfers implies that the locus of efficient contracts just differ by a non-state-contingent transfer between the two parties. Hence there is no loss of generality in focusing on a principal–agent problem with the principal offering a take-it-or-leave-it contract.

3.1. Statement-contingent contracts

A statement-contingent contract is characterized by a pair of vectors $(\mathbf{y},\mathbf{z}) \in Z \times Z$, where $\mathbf{z}$ is the statement-contingent output that the contract is designed to implement and $\mathbf{y}$ is the corresponding statement-contingent payment made by the principal to the agent. How these statement-contingent vectors are converted to the corresponding random variables faced by the two parties depends on how disputes are resolved. Notice here that a statement-contingent contract is contingent on the mutually agreed upon language S, *not* the state space $\Omega = S \times S$.[5]

For any vector $\mathbf{z} \in \mathbb{R}^{|S|}$, let $f_{\mathbf{z}}^{\mathrm{A}}$ (respectively, $f_{\mathbf{z}}^{\mathrm{P}}$) denote the random variable generated from $\mathbf{z}$ that is measurable with respect to S^{A} (respectively, S^{P}). That is, $f_{\mathbf{z}}^{\mathrm{A}}\left(s,s'\right) = \mathbf{z}_s$ (respectively, $f_{\mathbf{z}}^{\mathrm{P}}\left(s,s'\right) = \mathbf{z}_{s'}$) for every $\left(s,s'\right) \in \Omega$.

Definition 1. A random variable $\mathbf{z} \in \mathbb{R}^{|S|}$ *entails a non-null possibility of dispute* at $\left(s,s'\right)$, if $z_s \neq z_{s'}$ and $\left\{\left(s,s'\right)\right\} \notin \mathcal{N}$.

Essentially, a contract entails a non-null possibility of dispute if it might matter to the parties who have agreed to the contract.

3.1.1. War of attrition

A dispute may be modeled as a 'war of attrition' as in Grant et al. (2012, 2014). Recall that in the stationary mixed strategy Nash equilibrium of a war of attrition, at each stage a party must be indifferent between disputing the other party's interpretation or conceding. Hence it is as if the agent expects to have to provide the principal with the output level corresponding to the signal the principal receives and to be paid accordingly while the principal expects to be provided with the output level corresponding to the signal the agent receives and to pay accordingly. This is an extreme case which may apply when the parties do not have a clear probabilistic understanding of how disputes will be resolved and consequently, each expects the worst. The agent's state-contingent payoff is $f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{z}}^{\mathrm{P}} + f_{\mathbf{y}}^{\mathrm{P}}$ and the principal's is $f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{y}}^{\mathrm{A}}$.

The optimal statement-contingent contract in this setting is the solution to the following program:

$$\max_{(\mathbf{y},\mathbf{z}) \in Z \times Z} e^{\mathrm{P}}\left(f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{y}}^{\mathrm{A}}\right)$$

subject to

$$e^{\mathrm{A}}\left(f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{z}}^{\mathrm{P}} + f_{\mathbf{y}}^{\mathrm{P}}\right) \geqslant \underline{c} + x(\mathbf{z}) \tag{PC\_WA}$$

We shall refer to a solution to this program as an optimal war-of-attrition contract. We presume that there is a $(\mathbf{y},\mathbf{z}) \in Z \times Z$ that satisfies the participation constraint $e^{\mathrm{A}}\left(f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{z}}^{\mathrm{P}} + f_{\mathbf{y}}^{\mathrm{P}}\right) \geqslant \underline{c} + x(\mathbf{z})$. Since $Z \times Z$ is compact and $e^{\mathrm{A}}(\cdot), e^{\mathrm{P}}(\cdot)$ and $x(\mathbf{z})$ are continuous, the war-of-attrition program has a solution.[6]

Being able to make the payment to the agent conditional on the state eliminates the need to satisfy an incentive compatibility constraint. However, since the parties may disagree on which statement in the contract is operative, the parties are now subject to ambiguity and the associated costs.

There are other possible equilibrium outcomes for a dispute modeled as a war of attrition. Rather than exploring these we shall instead consider two legal procedures that result in polar opposite resolutions. The first always resolves disputes in accordance with the interpretation of the agent while the second does so in accordance with that of the principal. As we will see, both of these dominate the optimal war-of-attrition contract.

3.1.2. "Contra proferentem" doctrine

The contra-proferentem contract, is based on the doctrine of "*verba fortius accipiuntur contra proferentem*" (literally, "words are to be taken most strongly against him who uses them"), which is a rule of contractual interpretation which states that ambiguities in a contract should be construed against the party who drafted the contract. Since it is the principal who is choosing (that is, drafting) the contract, application of this doctrine entails any dispute being resolved in favor of the agent. Hence for the agent her state-contingent payoff is $f_{\mathbf{y}}^{\mathrm{A}}$ and for the principal it is $f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{y}}^{\mathrm{A}}$.

The optimal statement-contingent contract for our principal in this setting is the solution to:

$$\max_{\langle(\mathbf{y},\mathbf{z}) \in Z \times Z\rangle} e^{\mathrm{P}}\left(f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{y}}^{\mathrm{A}}\right)$$

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5 If the parties were able to contract on the full state space there would be no possibility of disputes and the contract would achieve the first best.

6 Since $Z \times Z$ is compact and there is a $(\mathbf{y},\mathbf{z}) \in Z \times Z$ that satisfies the participation constraint $e^{A}\left(f_{\mathbf{z}}^{A} - f_{\mathbf{z}}^{P} + f_{\mathbf{y}}^{P}\right) \geq \underline{c} + x(\mathbf{z})$, it follows by the continuity of $e^{A}(\cdot)$ and $x(\mathbf{z})$ that the constraint set $\left\{(\mathbf{y},\mathbf{z}) \in Z \times Z : e^{A}\left(f_{\mathbf{z}}^{A} - f_{\mathbf{z}}^{P} + f_{\mathbf{y}}^{P}\right) \geq \underline{c} + x(\mathbf{z})\right\}$ is compact. Then, since $e^{P}(\cdot)$ is continuous over the compact constraint set, the WA program has a solution by the Weierstrass Theorem.

subject to

$$e^{\mathrm{A}}\left(f_{\mathbf{y}}^{\mathrm{A}}\right) \geqslant \underline{c} + x(\mathbf{z}) \tag{PC\_CP}$$

We shall refer to a solution to this program as an optimal contra-proferentem contract. Under the presumption that there is a $(\mathbf{y},\mathbf{z}) \in Z \times Z$ that satisfies the participation constraint $e^{\mathrm{A}}\left(f_{\mathbf{y}}^{\mathrm{A}}\right) \geqslant \underline{c} + x(\mathbf{z})$, the contra proferentem program has a solution.

3.1.3. Mandatory arbitration

A mandatory-arbitration clause requires disputes to be resolved by an arbitration panel selected by the principal. This undoubtedly generates more resolutions of disputes that are more favorable to the principal.[7] For tractability, we model this under the polar assumption that any dispute is resolved in favor of the principal. Hence for the agent her state-contingent payoff is $f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{z}}^{\mathrm{P}} + f_{\mathbf{y}}^{\mathrm{P}}$ while for the principal it is $f_{\mathbf{z}}^{\mathrm{P}} - f_{\mathbf{y}}^{\mathrm{P}}$. One way to interpret this, is that there is a "spot" market where the agent can buy and sell as much of the output at the (normalized) price 1 to deliver to the principal what he expects, given his signal's realization.[8]

The optimal statement-contingent contract for our principal in this setting is the solution to:

$$\max_{\langle(\mathbf{y},\mathbf{z})\in Z\times Z\rangle} e^{\mathrm{P}}\left(f_{\mathbf{z}}^{\mathrm{P}} - f_{\mathbf{y}}^{\mathrm{P}}\right)$$

subject to

$$e^{\mathrm{A}}\left(f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{z}}^{\mathrm{P}} + f_{\mathbf{y}}^{\mathrm{P}}\right) \geqslant \underline{c} + x(\mathbf{z}) \tag{PC\_MA}$$

We shall refer to a solution to this program as an optimal mandatory-arbitration contract. Under the presumption that there is a $(\mathbf{y},\mathbf{z}) \in Z \times Z$ that satisfies the participation constraint $e^{\mathrm{A}}\left(f_{\mathbf{z}}^{\mathrm{A}} - f_{\mathbf{z}}^{\mathrm{P}} + f_{\mathbf{y}}^{\mathrm{P}}\right) \geqslant \underline{c} + x(\mathbf{z})$, this program also has a solution.

3.2. Output-contingent contracts

We now consider the case when the parties are unwilling or unable to make the contract contingent on statements in the common language S. Instead they contract on the basis of output which is assumed to be costlessly verifiable and hence unambiguous. This is the form of the standard moral hazard problem.[9]

An output-contingent contract is characterized by a function $y : [-\bar{z}, \bar{z}] \to \mathbb{R}$ and a vector $\mathbf{z} \in Z$, where $\mathbf{z}$ is the statement-contingent output that the contract is designed to implement. The vector $y \circ \mathbf{z} \in Z$ is the associated statement-contingent payment vector in which for each $s \in S$, $y(\mathbf{z}_s)$ is the payment that will be made by the principal to the agent in return for output $\mathbf{z}_s$. The optimal output-contingent contract is thus the solution to:

$$\max_{(y,\mathbf{z})} e^{\mathrm{P}}\left(f_{\mathbf{z}}^{\mathrm{A}} - f_{y\circ\mathbf{z}}^{\mathrm{A}}\right)$$

subject to

$$e^{\mathrm{A}}\left(f_{y\circ\mathbf{z}}^{\mathrm{A}}\right) \geqslant \underline{c} + x(\mathbf{z}) \tag{PC\_OC}$$

$$\mathbf{z} \in \operatorname{argmax}_{\hat{\mathbf{z}}\in Z}\ e^{\mathrm{A}}\left(f_{y\circ\hat{\mathbf{z}}}^{\mathrm{A}}\right) - x(\hat{\mathbf{z}})\,. \tag{IC\_OC}$$

We shall refer to the solution to this program as an optimal output-contingent contract. (PC_OC) is the standard participation constraint and (IC_OC) is the incentive-compatibility constraint. For the agent to be willing to accept the contract, (PC_OC) says that the agent's certainty equivalent of the statement-contingent payment vector $y \circ \mathbf{z}$ must be at least as great as the sum of her outside option $\underline{c}$ and the input cost of the production schedule $\mathbf{z}$. In addition, (IC_OC) notes that for the principal to expect the agent to plan to produce $\mathbf{z}$, the payment schedule y must be structured so as to make $\mathbf{z}$ a best choice for the agent from those technologically feasible production plans.

7 As Stone and Colvin (2015, p. 26) conclude in their report on abitration panels:

"By delegating dispute resolution to arbitration, the [Supreme] Court now permits corporations to write the rules that will govern their relationships with their workers and customers and design the procedures used to interpret and apply those rules when disputes arise. ... As one judge opined, these trends give corporations a 'get out of jail free' card for all potential transgressions."

8 Alternatively, one might model the situation as generating a relationship-specific product. In this case arbitration leads the agent to pay "damages" that makes the principal as well off as he would have been under his interpretation that statement s' has obtained. Furthermore, for a state (s, s') in which $\mathbf{z}_{s'} < \mathbf{z}_s$, the firm would only accept the output $\mathbf{z}_{s'}$, with the excess $\mathbf{z}_s - \mathbf{z}_{s'}$ 'lost'. In this case the agent's net payment would be $f_{\mathbf{y}}^{P} - \left[f_{\mathbf{z}}^{P} - f_{\mathbf{z}}^{A}\right]^{+}$, where $\left[f_{\mathbf{z}}^{P} - f_{\mathbf{z}}^{A}\right]^{+}(\omega) = \max\left\{f_{\mathbf{z}}^{P}(\omega) - f_{\mathbf{z}}^{A}(\omega), 0\right\}$.

9 For the representation of the technology used here, the problem is analyzed by Quiggin and Chambers (1998).

Table 1
Optimal contract programs.

	Principal's objective	IR constraint	IC constraint
WA	$e^{P}\left(f^{A}_{\mathbf{z}} - f^{A}_{\mathbf{y}}\right)$	$e^{A}\left(f^{A}_{\mathbf{z}} - f^{P}_{\mathbf{z}} + f^{P}_{\mathbf{y}}\right) \geqslant \underline{c} + x(\mathbf{z})$	No
CP	$e^{P}\left(f^{A}_{\mathbf{z}} - f^{A}_{\mathbf{y}}\right)$	$e^{A}\left(f^{A}_{\mathbf{y}}\right) \geqslant \underline{c} + x(\mathbf{z})$	No
MA	$e^{P}\left(f^{P}_{\mathbf{z}} - f^{P}_{\mathbf{y}}\right)$	$e^{A}\left(f^{A}_{\mathbf{z}} - f^{P}_{\mathbf{z}} + f^{P}_{\mathbf{y}}\right) \geqslant \underline{c} + x(\mathbf{z})$	No
OC	$e^{P}\left(f^{A}_{\mathbf{z}} - f^{A}_{y\circ\mathbf{z}}\right)$	$e^{A}\left(f^{A}_{y\circ\mathbf{z}}\right) \geqslant \underline{c} + x(\mathbf{z})$	Yes

4. Results

In this section, we present results comparing optimal contracts of various types in the presence of risk and ambiguity. Let C and C' denote two contracts that may be statement-contingent or output-contingent. We say that contract C *weakly dominates* contract C' if each party weakly prefers C to C'. Similarly, we say C *strictly dominates* C' if C weakly dominates C' and at least one party strictly prefers C to C'. The following Table 1 may help the reader to recall the differences between the various programs used to obtain our results.

First, consider what happens when there is no linguistic ambiguity. This corresponds to the conditional possibility event $S^{A}_{s} = S^{P}_{s} = \{s\}$ for all $s \in S$, that is, when one party sees s he is certain the other sees s as well. Without linguistic ambiguity, the optimal statement-contingent contracts are independent of the dispute resolution regime and are also first best efficient. This can be verified by observing in Table 1 that the principal's objectives and the IR constraints become equivalent across dispute resolution regimes and, by quasi-linearity of the certainty equivalents, the principal will maximize joint surplus. Optimal output-contingent contracts, on the other hand, typically involve the classic trade-off between risk and incentives, and are dominated by statement-contingent contracts.

More generally, in the presence of linguistic ambiguity, our results on domination can be summarized as follows. First, we show in Proposition 1 that when contra-proferentem contracts are available, output-contingent contracts will never be optimally chosen. Next, we dispense with war-of-attrition contracts by showing they are weakly dominated by contra-proferentem and mandatory-arbitration contracts (Proposition 2). In Proposition 3 we show that for the polar case of an ambiguity neutral principal, the optimal contra-proferentem contract fully insures the agent and dominates the optimal mandatory-arbitration contract. For any positive ambiguity aversion of the principal, however, we find that there are parameter configurations for the agent, namely those for which she is neither too ambiguity nor too risk averse, such that the optimal mandatory-arbitration contract dominates the optimal contra-proferentem contract (Proposition 4). These results motivate mixed dispute resolution regime contracts which are considered in Section 4.1.

Proposition 1. *The optimal contra-proferentem contract weakly dominates the optimal output-contingent contract.*

Proof. Let $\left(y', \mathbf{z}'\right)$ be an optimal output-contingent contract. Consider the contra-proferentem contract $(\mathbf{y}, \mathbf{z})$ in which $y_s = y\left(\mathbf{z}_s\right)$ and $\mathbf{z} = \mathbf{z}'$. This contract implements the optimal output-contingent contract and thus generates the same welfare to the agent and the principal. Since the principal no longer has to satisfy the IC constraint, the optimal contra-proferentem contract weakly dominates the optimal output-contingent contract. □

We utilize the following basic lemma in the proofs of the propositions below. The proof of the basic lemma makes use of the assumption (9). This assumption allows us to focus on how differences in ambiguity and risk attitudes between the principal and agent affect the optimality of a dispute resolution scheme.

Lemma 1. *For any pair of vectors* $(\mathbf{y}, \mathbf{z}) \in Z \times Z$,

1. $e^{A}\left(f^{A}_{\mathbf{y}}\right) \geqslant e^{A}\left(f^{A}_{\mathbf{z}} - f^{P}_{\mathbf{z}} + f^{P}_{\mathbf{y}}\right)$*, with strict inequality whenever* $\mathbf{z}$ *entails a non-null possibility of dispute at some* $\left(s, s'\right) \in \Omega$*;*
2. $e^{P}\left(f^{P}_{\mathbf{z}}\right) \geqslant e^{P}\left(f^{A}_{\mathbf{z}}\right)$*, with strict inequality whenever* $\mathbf{z}$ *entails a non-null possibility of dispute at some* $\left(s, s'\right)$ *for which* $\alpha^{P}_{s'} > 0$.

Proof. (1): From (9) we have $\mu^{A}_{S^{A}}(s) = \mu^{A}_{S^{P}}(s)$ for all $s \in S$, and from (3) we have $\alpha^{A}_{s} \geqslant \alpha^{A}_{0}$ for all s. Thus, by Jensen's inequality we have $e^{A}\left(f^{A}_{\mathbf{y}}\right) \geqslant e^{A}\left(f^{P}_{\mathbf{y}}\right)$. Since $f^{A}_{\mathbf{z}} - f^{P}_{\mathbf{z}} + f^{P}_{\mathbf{y}}$ is a mean-preserving spread of $f^{P}_{\mathbf{y}}$, Jensen's inequality also yields $e^{A}\left(f^{P}_{\mathbf{y}}\right) \geqslant e^{A}\left(f^{A}_{\mathbf{z}} - f^{P}_{\mathbf{z}} + f^{P}_{\mathbf{y}}\right)$ with strict inequality whenever $\mathbf{z}$ entails a non-null possibility of dispute at some $\left(s, s'\right) \in \Omega$.

(2): From (9) we have $\mu^{P}_{S^{A}}(s) = \mu^{P}_{S^{P}}(s)$ for all $s \in S$, and from (6) we have $\alpha^{P}_{s} \geqslant 0$ for all s. Thus, by Jensen's inequality, we obtain $e^{P}\left(f^{P}_{\mathbf{z}}\right) \geqslant e^{P}\left(f^{A}_{\mathbf{z}}\right)$ with strict inequality whenever $\mathbf{z}$ entails a non-null possibility of dispute at some $\left(s, s'\right)$ for which $\alpha^{P}_{s'} > 0$. □

The next proposition suggests that war-of-attrition contracts can be dispensed with if optimal contra-proferentem or mandatory-arbitration contracts are available. This is perhaps not surprising since both parties are subject to ambiguity under wars of attrition, while one party is spared under contra proferentem and mandatory arbitration.

Proposition 2. *The optimal contra-proferentem contract and the optimal mandatory-arbitration contract each weakly dominate the optimal war-of-attrition contract.*

Proof. Let $(\mathbf{y}, \mathbf{z})$ be the optimal war-of-attrition contract. Consider the contra-proferentem contract $(\mathbf{y}, \mathbf{z})$. By construction, the principal is indifferent between the two contracts. However, by part 1 of Lemma 1, $e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{y}}\right) \geqslant e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{z}} - f^{\mathrm{P}}_{\mathbf{z}} + f^{\mathrm{P}}_{\mathbf{y}}\right)$.

Similarly, we can use part 2 of Lemma 1 to show that the optimal mandatory-arbitration contract weakly dominates the optimal war-of-attrition contract. □

Based on these findings, we focus attention in what follows on the contra-proferentem and mandatory-arbitration contracts. The contra-proferentem contract has the advantage of putting no ambiguity on the agent. If the principal is ambiguity neutral in the sense that $\alpha^{\mathrm{P}}_s = 0$ for all $s \in S$, then applying the contra proferentem doctrine will dominate requiring mandatory arbitration as we show in the next proposition. In part (4) we show that the optimal contra-proferentem strictly dominates any mandatory-arbitration contract that entails a non-null possibility of dispute.

Proposition 3. *Suppose the principal is ambiguity neutral in the sense that $\alpha^{\mathrm{P}}_s = 0$ for all $s \in S$. (1) The optimal mandatory-arbitration and war-of-attrition contracts are equivalent; (2) The optimal contra-proferentem contract fully insures the agent; (3) The optimal contra-proferentem contract weakly dominates any mandatory-arbitration contract; (4) The optimal contra-proferentem contract strictly dominates any mandatory-arbitration contract $\left(\mathbf{y}', \mathbf{z}'\right)$ for which $\mathbf{z}'$ entails a non-null possibility of dispute at some $(s, s') \in \Omega$.*

Proof. (1) Since the principal is ambiguity neutral, it follows that $e^{\mathrm{P}}\left(f^{\mathrm{P}}_{\mathbf{z}} - f^{\mathrm{P}}_{\mathbf{y}}\right) = e^{\mathrm{P}}\left(f^{\mathrm{A}}_{\mathbf{z}} - f^{\mathrm{A}}_{\mathbf{y}}\right)$ for any contract $(\mathbf{y}, \mathbf{z})$. Consequently, the war-of-attrition program and the mandatory-arbitration program are equivalent.

(2) Consider any contra-proferentem contract $(\mathbf{y}, \mathbf{z})$ that does not fully insure the agent, that is, $y_s \neq y_{s'}$ for some states $s, s' \in S$. By the risk aversion of the agent, it follows that $\sum_{s\in S} \mu(s) y_s = \sum_{s\in S} \mu^{\mathrm{A}}_{S^{\mathrm{A}}} y_s > e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{y}}\right)$. Consider the contra-proferentem contract $(\mathbf{y}', \mathbf{z}')$ with $y'_s = e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{y}}\right)$ for all $s \in S$ and with $\mathbf{z}' = \mathbf{z}$. The agent will be indifferent between $(\mathbf{y}, \mathbf{z})$ and $(\mathbf{y}', \mathbf{z}')$. Since by (9), $\mu^{\mathrm{A}}_{S^{\mathrm{A}}}(s) = \mu^{\mathrm{P}}_{S^{\mathrm{A}}}(s) = \mu(s)$, the contract $(\mathbf{y}', \mathbf{z}')$ involves a smaller expected wage payment which makes the principal strictly better off. Consequently, $(\mathbf{y}, \mathbf{z})$ cannot be an optimal contra-proferentem contract.

(3) This follows from (1) and Proposition 2.

(4) Let $(\mathbf{y}, \mathbf{z})$ be an optimal mandatory-arbitration contract for which $\mathbf{z}$ entails a non-null possibility of dispute at state $(s, s') \in \Omega$. Consider the contra-proferentem contract $(\mathbf{y}, \mathbf{z})$. By Lemma 1 (2) the principal is no worse off. By Lemma 1 (1) the agent is strictly better off. □

For an ambiguity averse principal the following provides sufficient conditions for mandatory arbitration to dominate the contra proferentem doctrine.

Proposition 4. *If the agent's risk and ambiguity aversions are sufficiently low, the optimal mandatory-arbitration contract will strictly dominate any optimal contra proferentem contract $(\mathbf{y}, \mathbf{z})$ for which the random variable $\mathbf{z} - \mathbf{y}$ entails a non-null possibility of dispute at some $(s, s') \in \Omega$ where $\alpha^{\mathrm{P}}_{s'} > 0$.*

Proof. Let $(\mathbf{y}, \mathbf{z})$ be any optimal contra-proferentem contract for which the random variable $\mathbf{z} - \mathbf{y}$ entails a possibility of dispute at a non-null state $(s, s') \in \Omega$. Consider the same contract under mandatory arbitration. Since $\alpha^{\mathrm{P}}_{s'} > 0$, it follows from part 2 of Lemma 1 that $e^{\mathrm{P}}\left(f^{\mathrm{P}}_{\mathbf{z}} - f^{\mathrm{P}}_{\mathbf{y}}\right) - e^{\mathrm{P}}\left(f^{\mathrm{A}}_{\mathbf{z}} - f^{\mathrm{A}}_{\mathbf{y}}\right) > \varepsilon$ for some $\varepsilon > 0$. Since $f^{\mathrm{A}}_{\mathbf{y}}$ and $f^{\mathrm{A}}_{\mathbf{z}} - f^{\mathrm{P}}_{\mathbf{z}} + f^{\mathrm{P}}_{\mathbf{y}}$ have the same mean, it follows from the continuity of the certainty equivalents that as we reduce $\alpha^{\mathrm{A}}_{s''}$ $(\geqslant \alpha^{\mathrm{A}}_0)$ toward zero for all $s'' \in S$, the difference $e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{y}}\right) - e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{z}} - f^{\mathrm{P}}_{\mathbf{z}} + f^{\mathrm{P}}_{\mathbf{y}}\right)$ tends to zero. In particular, there exists $\delta > 0$ such that $e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{y}}\right) - e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{z}} - f^{\mathrm{P}}_{\mathbf{z}} + f^{\mathrm{P}}_{\mathbf{y}}\right) < \varepsilon$ whenever $\alpha^{\mathrm{A}}_{s''} < \delta$ for all $s'' \in S$. □

4.1. "Mixed" contra proferentem/mandatory-arbitration contract

The results of the previous subsection, in particular, Propositions 3 and 4 indicate that the optimal choice of mandatory arbitration or contra proferentem as a means of managing dispute resolution depends upon the risk and ambiguity parameters of the parties to the contract. Since the ambiguity parameters are statement and party dependent, it is plausible that allowing a contract to condition the type of dispute resolution (mandatory arbitration or contra proferentem) on statements, or subsets of statements, could improve the welfare of both parties. However, to be able to implement such contracts, there would need to be no ambiguity as to whether or not the subset of statements holds.

It seems natural for a subset $E \subseteq S$ of statements to be deemed an *unambiguous event* whenever each party understands that if the realization of their signal is in E then so will the realization of the other party's signal. Hence, there can be no dispute between the parties about that event obtaining. Adapting Grant et al. (2012), we formally define the set of unambiguous events as follows.

Definition 2. The set of *unambiguous* events $\mathcal{E}_{\text{U}} \subseteq 2^S$ is given by

$$\mathcal{E}_{\text{U}} = \left\{ E \subseteq S : \bigcup_{s \in E} S_s^{\text{A}} = \bigcup_{s \in E} S_s^{\text{P}} = E \right\}.$$

In Grant et al. (2012, Lemma 1), we established that $\mathcal{E}_{\text{U}}$ is an algebra under conditions that correspond here to (1) and (2). We thus allow the two parties to condition which dispute resolution regime operates on any unambiguous event. We refer to such a contract as a *mixed dispute-resolution-regime contract.* A mixed dispute-resolution-regime contract is characterized by a pair $(\mathbf{y}, \mathbf{z}) \in Z \times Z$, and an event $E \in \mathcal{E}_{\text{U}}$. The vectors $\mathbf{y}$ and $\mathbf{z}$ are as before, while E denotes the event in which the agent has agreed to set aside the *contra proferentem* doctrine and instead have any dispute resolved by *mandatory arbitration*.

To specify the program for a principal choosing a mixed dispute-resolution-regime contract it is convenient to introduce the following notation. For any vector $\mathbf{z} \in Z$ and any $E \in \mathcal{E}_{\text{U}}$ let $\left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{A}}\right)$ denote the random variable

$$\left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{A}}\right)\left(s, s'\right) = \begin{cases} \mathbf{z}_{s'} & \text{if } s \in E \\ \mathbf{z}_s & \text{if } s \notin E. \end{cases}$$

This random variable agrees with $f_{\mathbf{z}}^{\text{P}}$ on $E \times S$ and with $f_{\mathbf{z}}^{\text{A}}$ on $S \backslash E \times S$.[10]

We shall associate with a mixed dispute-resolution-regime contract $(\mathbf{y}, \mathbf{z}, E)$ the random variable $\left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{A}}\right)$.

The optimal statement-contingent contract for our principal in the mixed dispute-resolution-regime setting is the solution to the following program:

$$\max_{\langle (\mathbf{y},\mathbf{z}) \in Z \times Z, E \in \mathcal{E}_{\text{U}} \rangle} e^{\text{P}} \left(\left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{A}}\right) - \left(f_{\mathbf{y}}^{\text{P}}\right)_E \left(f_{\mathbf{y}}^{\text{A}}\right) \right)$$

subject to

$$e^{\text{A}} \left(f_{\mathbf{z}}^{\text{A}} - \left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{A}}\right) + \left(f_{\mathbf{y}}^{\text{P}}\right)_E \left(f_{\mathbf{y}}^{\text{A}}\right) \right) \geq \underline{c} + x(\mathbf{z}). \qquad \text{(PC\_MDRR)}$$

Optimal mixed dispute-resolution-regime contracts will always weakly dominate optimal mandatory-arbitration and optimal contra-proferentem contracts since S is always an unambiguous event. The insights of Propositions 3 and 4 can be applied to an unambiguous event $E \neq S$ to find situations where mixed contracts strictly dominate. The following corollary follows directly from part (4) of Proposition 3.

Corollary 1. *Let $E \neq S$ be an unambiguous event over which the principal is ambiguity neutral. If the optimal mandatory-arbitration contract $(\mathbf{y}, \mathbf{z})$ weakly dominates the optimal contra-proferentem contract and $\mathbf{z}$ entails a non-null possibility of dispute at some state $(s, s') \in E \times E$, then the optimal mixed dispute-resolution-regime contract strictly dominates both the optimal mandatory-arbitration and the optimal contra-proferentem contracts.*

5. The linear exponential (bivariate) normal case: an illustration

In this section, we consider an example where the signal space is the entire real line $\mathbb{R}$. We take $\Omega = S^{\text{A}} \times S^{\text{P}} = \mathbb{R} \times \mathbb{R}$ with generic element $\omega = (s^{\text{A}}, s^{\text{P}})$.[11] With appropriate modifications, the framework developed above for the finite statement space can be extended to the continuous signal case. For this example, we presume a common prior $\mu(s, s')$ having a (symmetric) bivariate normal distribution with mean $(0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix}, \quad \text{where } \rho \in (0, 1) \text{ and } \sigma^2 > 0.$$

Notice that, under this signal structure, when the realization of one individual's signal is s, the other individual could see anything in $S = \mathbb{R}$. As a consequence, any payments that vary with the state will entail a non-null possibility of dispute. We take the technologically feasible statement contingent output vectors to be those in the set $\{(z + s)_{s \in S} \colon z \geqslant 0\}$. That is, they are restricted to be those that can be obtained by adding some noise to an (expected) output level $z \geqslant 0$. We identify each (technically feasible) statement contingent output vector $\mathbf{z}$ with its expected output level $z \geqslant 0$ and associate with it,

[10] Alternatively, we might consider the random variable defined as:

$$\left(g_{\mathbf{z}}^{\text{P}}\right)_E \left(g_{\mathbf{z}}^{\text{A}}\right)\left(s, s'\right) = \begin{cases} \mathbf{z}_{s'} & \text{if } s' \in E \\ \mathbf{z}_s & \text{if } s' \notin E. \end{cases}$$

Notice that $\left(g_{\mathbf{z}}^{\text{P}}\right)_E \left(g_{\mathbf{z}}^{\text{A}}\right)$ agrees with $\left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{A}}\right)$ on $E \times E$ and on $S \backslash E \times S \backslash E$. Now, since $E \in \mathcal{E}_{\text{U}}$, it follows that $E \times S \backslash E$ and $S \backslash E \times E$ are null for both parties. Hence we have $e^{\text{A}}\left(\left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{P}}\right)\right) = e^{\text{A}}\left(\left(g_{\mathbf{z}}^{\text{P}}\right)_E \left(g_{\mathbf{z}}^{\text{A}}\right)\right)$ and $e^{\text{P}}\left(\left(f_{\mathbf{z}}^{\text{P}}\right)_E \left(f_{\mathbf{z}}^{\text{A}}\right)\right) = e^{\text{P}}\left(\left(g_{\mathbf{z}}^{\text{P}}\right)_E \left(g_{\mathbf{z}}^{\text{A}}\right)\right)$, for all $\mathbf{z} \in \mathbb{R}^{|S|}$.

[11] It is standard to use the Cartesian product of the Borel sets, e.g., $B_\omega = B_{(s,s')} = (-\infty, s] \times (-\infty, s']$ and close it under complementation and countable unions to get the minimal sigma algebra.

its input requirement $x(\mathbf{z}) = z^2/(2\gamma)$, where $\gamma > 0$. Having chosen $\mathbf{z}$, when state $\omega = \left(s^{\mathrm{A}}, s^{\mathrm{P}}\right)$ occurs, the agent realizes an output level $f^{\mathrm{A}}_{\mathbf{z}}\left(s^{\mathrm{A}}, s^{\mathrm{P}}\right) = z + s^{\mathrm{A}}$ while the principal expects an output $f^{\mathrm{P}}_{\mathbf{z}}\left(s^{\mathrm{A}}, s^{\mathrm{P}}\right) = z + s^{\mathrm{P}}$.

Both the agent and principal have the infinite state space analog of the RCARA preferences defined in section 2.2. Thus, the agent's preferences can be characterized by the tuple $(\alpha^{\mathrm{A}}_0, (\alpha^{\mathrm{A}}_s)_{s\in S})$ where, for simplicity, we take $\alpha^{\mathrm{A}}_s = \alpha^{\mathrm{A}}_1$ $(\geqslant \alpha^{\mathrm{A}}_0 > 0)$ for all $s \in S$. Since the principal is risk neutral, his preferences can be characterized by his ambiguity parameters $(\alpha^{\mathrm{P}}_s)_{s\in S}$ where, again for simplicity, we take $\alpha^{\mathrm{P}}_s = \alpha^{\mathrm{P}}_1 \geqslant 0$ for all $s \in S$.

If we allow general contracts, then as is well-known (for example, Mirrlees, 1974), since the certainty equivalent function of the agent is quasi-linear in transfers and transfers are unbounded there is no optimal solution. The first-best (informationally unconstrained) solution can be approximated arbitrarily closely with a two-part output contract. As a consequence of this, in what follows, we focus on linear contracts.

We first consider linear statement-contingent contracts that can be characterized by a triple (z, λ, k) where z is an expected output level and λ (respectively, k) is a per unit signal (respectively, fixed) payment to be made from the principal to the agent.

5.1. War of attrition

Recall that in a war-of-attrition setting, each individual expects the contract to be enforced in terms of what the other sees. That is, for the linear contract (z, λ, k), the agent first sinks the input requirement $z^2/(2\gamma)$. Then if state $\left(s, s'\right) \in \Omega$ obtains, the payoff to the principal in the stationary war-of-attrition equilibrium is $(z + s) - \lambda s - k$ while the agent's payoff is $(z + s) - \left(z + s'\right) + \lambda s' + k - z^2/(2\gamma)$. Hence, the principal views the linear contract (z, λ, k) as inducing the random variable $f^{\mathrm{A}}_{\mathbf{z}} - \lambda\left(f^{\mathrm{A}}_{\mathbf{z}} - z\right) - k$, while the agent perceives views the contract as inducing the random variable $\left(f^{\mathrm{A}}_{\mathbf{z}} - f^{\mathrm{P}}_{\mathbf{z}}\right) + \lambda\left(f^{\mathrm{P}}_{\mathbf{z}} - z\right) + k - z^2/(2\gamma)$. Since RCARA preferences are quasi-linear with respect to non-state contingent transfers, the optimal (linear) contract should maximize the total surplus of the principal and agent subject to the agent's willingness to participate. That is, it is the solution to:

$$\max_{z,\lambda,k} e^{\mathrm{P}}\left((1-\lambda)\, f^{\mathrm{A}}_{\mathbf{z}} + \lambda z - k\right) + e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{z}} - (1-\lambda)\, f^{\mathrm{P}}_{\mathbf{z}} - \lambda z + k - \frac{z^2}{2\gamma}\right)$$

$$\text{subject to } e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{z}} - (1-\lambda)\, f^{\mathrm{P}}_{\mathbf{z}} - \lambda z + k - \frac{z^2}{2\gamma}\right) \geqslant \underline{c}$$

From the properties of the certainty equivalent functions of the RCARA preferences of the principal and the agent and given the distributions of $f^{\mathrm{A}}_{\mathbf{z}}$ and $f^{\mathrm{P}}_{\mathbf{z}}$, it readily follows that:

$$e^{\mathrm{P}}\left((1-\lambda)\, f^{\mathrm{A}}_{\mathbf{z}} + \lambda z - k\right) = z - \left(1-\rho^2\right)(1-\lambda)^2\,\alpha^{\mathrm{P}}_1\frac{\sigma^2}{2} - k \tag{10}$$

$$e^{\mathrm{A}}\left(f^{\mathrm{A}}_{\mathbf{z}} - (1-\lambda)\, f^{\mathrm{P}}_{\mathbf{z}} - \lambda z + k - \frac{z^2}{2\gamma}\right) = -\left[\left(1-\rho^2\right)(1-\lambda)^2\,\alpha^{\mathrm{A}}_1 + \lambda^2\alpha^{\mathrm{A}}_0\right]\frac{\sigma^2}{2} + k - \frac{z^2}{2\gamma} \tag{11}$$

Summing (10) and (11), we find that the principal's program for selecting the optimal z and λ may be re-expressed as

$$\max_{z,\lambda} z - \left[\left(1-\rho^2\right)(1-\lambda)^2\left(\alpha^{\mathrm{P}}_1 + \alpha^{\mathrm{A}}_1\right) + \lambda^2\alpha^{\mathrm{A}}_0\right]\frac{\sigma^2}{2} - \frac{z^2}{2\gamma} \tag{12}$$

Differentiating (12) with respect to z and λ, setting the resulting first order conditions equal to zero, solving for λ and z, and then finding k from the agent's participation constraint yields:

$$z^{\mathrm{WA}} = \gamma,$$

$$\lambda^{\mathrm{WA}} = \frac{\left(1-\rho^2\right)\left(\alpha^{\mathrm{P}}_1 + \alpha^{\mathrm{A}}_1\right)}{\left(1-\rho^2\right)\left(\alpha^{\mathrm{P}}_1 + \alpha^{\mathrm{A}}_1\right) + \alpha^{\mathrm{A}}_0},$$

$$k^{\mathrm{WA}} = \underline{c} + \frac{\gamma}{2} + \left[\left(1-\lambda^{\mathrm{WA}}\right)^2\left(1-\rho^2\right)\alpha^{\mathrm{A}}_1 + \left(\lambda^{\mathrm{WA}}\right)^2\alpha^{\mathrm{A}}_0\right]\frac{\sigma^2}{2}$$

Since $\alpha^{\mathrm{A}}_1 \geqslant \alpha^{\mathrm{A}}_0 > 0$, $\alpha^{\mathrm{P}}_1 \geqslant 0$ and $\rho^2 < 0$ it follows that $0 < \lambda^{\mathrm{WA}} < 1$. The optimal contract is more "high-powered", that is, the bigger is λ^{WA}: (*i*) the less the two parties' signals are correlated (that is, the smaller is ρ); (*ii*) the more ambiguity averse is either party (that is, the bigger is α^{A}_1 or α^{P}_1); and, (*iii*) the less risk averse is the agent (that is, the smaller is α^{A}_0). The fixed payment k^{WA} covers the agent's outside option, the cost of production and the cost of risk and ambiguity to the agent.

5.2. "Contra proferentem" doctrine

If the contra proferentem doctrine applies then the principal views the linear contract (z, λ, k) as inducing the random variable $f_{\mathbf{z}}^{A} - \lambda\left(f_{\mathbf{z}}^{A} - z\right) - k$, while the agent perceives it as delivering her the random variable $\lambda\left(f_{\mathbf{z}}^{A} - z\right) + k - z^2/(2\gamma)$. The certainty equivalent for the principal is as in (10). Since the agent is not subject to ambiguity, her certainty equivalent of the random variable she faces is given by:

$$e^{A}\left(\lambda f_{\mathbf{z}}^{A} + k\right) = -\lambda^2 \alpha_0^{A} \frac{\sigma^2}{2} + k - \frac{z^2}{2\gamma} \tag{13}$$

Thus the principal's problem reduces to:

$$\max_{z,\lambda,k} z - \left[(1-\lambda)^2\left(1-\rho^2\right)\alpha_1^{P} + \lambda^2\alpha_0^{A}\right]\frac{\sigma^2}{2} - \frac{z^2}{2\gamma} \tag{14}$$
$$\text{subject to } k \geqslant \underline{c} + \frac{z^2}{2\gamma} + \lambda^2\alpha_0^{A}\frac{\sigma^2}{2},$$

yielding the solution:

$$z^{CP} = \gamma,$$
$$\lambda^{CP} = \frac{\left(1-\rho^2\right)\alpha_1^{P}}{\left(1-\rho^2\right)\alpha_1^{P} + \alpha_0^{A}},$$
$$k^{CP} = \underline{c} + \frac{\gamma}{2} + \left[\left(\lambda^{CP}\right)^2\alpha_0^{A}\right]\frac{\sigma^2}{2}$$

Since $\alpha_0^{A} > 0$, $\alpha_1^{P} \geqslant 0$ and $\rho^2 < 1$ it follows that $0 \leqslant \lambda^{CP} < 1$, with $\lambda^{CP} = 0$ if and only if $\alpha_1^{P} = 0$. And, as was the case for the war-of-attrition, the contract becomes more "high-powered" (that is, the higher is λ^{CP}) the less the two parties's signals are correlated, the more ambiguity averse is the principal and the less risk averse is the agent. But since the contra-proferentem contract does not subject the agent to any ambiguity, her ambiguity parameter α_1^{A} plays no part in the solution.

Comparing the total surplus expressions (12) for the war-of-attrition contract and (14) for the contra-proferentem contract, we see immediately that the later strictly dominates: simply consider the contra-proferentem contract that mimics the optimal λ^{WA}. This contract will generate an additional surplus in the amount of $\left[\left(1-\rho^2\right)\left(1-\lambda^{WA}\right)^2\alpha_1^{A}\right]\sigma^2/2 > 0$. This surplus comes from avoiding the ambiguity costs on the agent which are positive since the agent is ambiguity averse $\left(\alpha_1^{A} > 0\right)$.

5.3. Mandatory arbitration

Recall, in the mandatory arbitration scenario, disputes are resolved by an arbitration panel selected by the principal with any resulting dispute being resolved in favor of the principal. Thus the principal views the linear contract (z, λ, k) as inducing the random variable $f_{\mathbf{z}}^{P} - \lambda\left(f_{\mathbf{z}}^{P} - z\right) - k$, while the agent perceives it as inducing the random variable $f_{\mathbf{z}}^{A} - (1-\lambda) f_{\mathbf{z}}^{P} - \lambda z + k - z^2/(2\gamma)$. In this case, the agent is the only one subject to ambiguity and the certainty equivalent of the random variable she perceives the contract will deliver her is given by (11), as in the war-of-attrition. Since the principal is risk neutral and now faces no ambiguity cost, his certainty equivalent becomes simply the expected value of $(1-\lambda) f_{\mathbf{z}}^{P} - k$. That is,

$$e^{P}\left((1-\lambda) f_{\mathbf{z}}^{P} + \lambda z - k\right) = z - k. \tag{15}$$

Thus the principal's problem reduces to:

$$\max_{z,\lambda,k} z - \left[(1-\lambda)^2\left(1-\rho^2\right)\alpha_1^{A} + \lambda^2\alpha_0^{A}\right]\frac{\sigma^2}{2} - \frac{z^2}{2\gamma} \tag{16}$$
$$\text{subject to } k \geqslant \underline{c} + \frac{z^2}{2\gamma} + \left[(1-\lambda)^2\left(1-\rho^2\right)\alpha_1^{A} + \lambda^2\alpha_0^{A}\right]\frac{\sigma^2}{2},$$

yielding the solution:

$$z^{MA} = \gamma,$$
$$\lambda^{MA} = \frac{\left(1-\rho^2\right)\alpha_1^{A}}{\left(1-\rho^2\right)\alpha_1^{A} + \alpha_0^{A}},$$

$$k^{\mathrm{MA}} = \underline{c} + \frac{\gamma}{2} + \left[\left(1-\lambda^{\mathrm{MA}}\right)^2\left(1-\rho^2\right)\alpha_1^{\mathrm{A}} + \left(\lambda^{\mathrm{MA}}\right)^2\alpha_0^{\mathrm{A}}\right]\frac{\sigma^2}{2}$$

Since $\alpha_1^{\mathrm{A}} \geqslant \alpha_0^{A} > 0$ and $\rho^2 < 1$ it follows that $0 < \lambda^{\mathrm{MA}} < 1$. The contract is more high-powered (that is, the higher is λ^{MA}) the less the two parties's signals are correlated, and the more ambiguity averse (respectively, less risk averse) is the agent. But now, since the principal is not subjected to ambiguity, his ambiguity aversion parameter, α_1^{P}, plays no role in determining the solution.

Comparing the surplus from the optimal mandatory-arbitration contract with that from the optimal war-of-attrition contract, we readily see that the former strictly dominates the later if (and only if) the principal is ambiguity averse $\left(\alpha_1^{\mathrm{P}} > 0\right)$. This strict dominance is the result of the mandatory arbitration scheme avoiding ambiguity costs to the principal. When the principal is ambiguity neutral $\left(\alpha_1^{\mathrm{P}} = 0\right)$, however, since there are no ambiguity costs to the principal, the optimal mandatory-arbitration and the optimal war-of-attrition contracts coincide.

Determining which of the optimal contra-proferentem contract and the optimal mandatory-arbitration contract dominates is particularly straightforward in this example. It reduces to a simple comparison between the respective ambiguity costs of the two forms of contract. In short, the optimal contra-proferentem contract strictly dominates the optimal mandatory-arbitration contract if and only if $\alpha_1^{\mathrm{A}} > \alpha_1^{\mathrm{P}}$. When the ambiguity attitudes of the individuals coincide $\left(\alpha_1^{\mathrm{P}} = \alpha_1^{\mathrm{A}}\right)$, the two schemes generate exactly the same surplus.[12]

5.4. Output-contingent contracts

If the parties are unwilling or unable to make the contract contingent on statements in the common language S and instead contract on the basis of output, then the contract is characterized by a triple (z, λ, k), where z and k are as before, but λ is now a per unit *output* payment to be made from the principal to the agent. The principal views the contract as delivering him the random variable $(1-\lambda)\, f_{\mathbf{z}}^{\mathrm{A}} - k$ while the agent perceives it as delivering her the random variable $\lambda f_{\mathbf{z}}^{\mathrm{A}} + k$. The contract imposes no ambiguity on the agent, however, there is now an added incentive compatibility constraint for the agent. Thus the problem facing the principal may be expressed as follows:

$$\max_{z,\lambda,k} z - \left[(1-\lambda)^2\left(1-\rho^2\right)\alpha_1^{\mathrm{P}} + \lambda^2\alpha_0^{\mathrm{A}}\right]\frac{\sigma^2}{2} - \frac{z^2}{2\gamma}$$

$$\text{subject to } z \in \operatorname{argmax}_{\hat{z}}\, \lambda\hat{z} - \lambda^2\alpha_0^{\mathrm{A}}\frac{\sigma^2}{2} + k - \frac{\hat{z}^2}{2\gamma},$$

$$\text{and } k \geqslant \underline{c} + \frac{z^2}{2\gamma} - \lambda z + \lambda^2\alpha_0^{\mathrm{A}}\frac{\sigma^2}{2}.$$

The first (incentive capability) constraint implies that the agent will choose $z = \gamma\lambda$. Substituting this into the objective function, the principal's problem for determining the optimal λ becomes:

$$\max_{\lambda} \gamma\lambda - \left[(1-\lambda)^2\left(1-\rho^2\right)\alpha_1^{\mathrm{P}} + \lambda^2\alpha_0^{\mathrm{A}}\right]\frac{\sigma^2}{2} - \frac{\gamma\lambda^2}{2}. \tag{17}$$

Differentiating expression (17) with respect to λ, setting the resulting first order condition equal to zero, and using the second (participation) constraint to find k, yields

$$z^{\mathrm{OC}} = \gamma\lambda^{\mathrm{OC}},$$

$$\lambda^{\mathrm{OC}} = \frac{\gamma + \sigma^2\left(1-\rho^2\right)\alpha_1^{\mathrm{P}}}{\gamma + \sigma^2\left[\left(1-\rho^2\right)\alpha_1^{\mathrm{P}} + \alpha_0^{\mathrm{A}}\right]},$$

$$k^{\mathrm{OC}} = \underline{c} - \frac{\left(\lambda^{\mathrm{OC}}\right)^2\left(\gamma - \sigma^2\alpha_0^{\mathrm{A}}\right)}{2}$$

Since $\alpha_0^{\mathrm{A}} > 0$, $\alpha_1^{\mathrm{P}} \geqslant 0$, $\rho^2 < 1$ and $\gamma > 0$, it follows that $0 < \lambda^{\mathrm{OC}} < 1$. Thus, the optimal output contingent contract generates an inefficiently low (expected) output level z. The same λ^{OC} can be used in the contra proferentem scheme to generate an efficient output level with exactly the same cost of risk and ambiguity. Thus, the total surplus must be higher under the optimal contra-proferentem contract.

[12] With respect to mixed contracts and Proposition 5, however, we do not find any improvements in this example since the only unambiguous event is S.

6. Concluding comments

Ambiguity is most naturally considered in an interpersonal context. The standard theory of subjective probability and the associated consistency requirements seem most convincing when applied to beliefs about states of nature derived from direct observation and introspection. By contrast, information derived from the statements of others is typically ambiguous. Even in the absence of any intent to deceive, we can never be sure that we have taken the meaning intended by others, or communicated the message we intended to send. In the present paper, we have shown that our model of two-stage RCARA preferences provides a tractable way to handle this problem in a principal–agent contracting framework. In particular we have shown that the outcome under linguistic ambiguity is weakly Pareto-dominated by the first best outcome that can be realized in the absence of ambiguity. Our results confirm the received view that in drawing up contracts, precise and unambiguous drafting is mutually beneficial.

There are situations, however, where ambiguity regarding the details of an agreement is often held to be advantageous. We conjecture that such situations may arise where the parties are unaware of some future possibilities and understand this fact. In future work, we plan to explore the formulation of mutually beneficial agreements in the presence of both linguistic ambiguity and bounded awareness.

Appendix A. An axiomatization of RCARA preferences

In this section we show for the case where the state space is finite that the RCARA representation is characterized by the three properties in Section 2 that were shown to be satisfied by the preferences of the principal and the agent.[13]

We begin with the larger class of preferences for which every RCARA preference relation belongs, namely the ones in which every own-signal realization is essential and the relation admits a continuous certainty equivalent representation. In behavioral terms these are preference relations in which, for each random variable, there is a *unique* constant random variable with respect to which the individual is indifferent. Equivalently, this can be formally stated as follows.

Definition 3. A preference relation $\succsim^i$ on $\mathcal{F}$ admits a *continuous certainty equivalent representation* if there exists a continuous (in the topology of pointwise convergence) function $e : \mathcal{F} \to \mathbb{R}$, such that for all pairs of random variables $f, f' \in \mathcal{F}$,

$$e(f) \geqslant e\left(f'\right) \text{ if and only if } f \succsim^i f'.$$

As is standard in the analysis of principal–agent problems and other *partial equilibrium* settings, our focus above has been on individuals for whom wealth has no effect on their attitudes toward risk and ambiguity. That is, no matter what the individual's current state-contingent endowment is, adding a state-independent amount of wealth leaves unaffected the set of state-contingent bets she would be willing to accept. In the standard expected utility framework, these are preferences for which risk attitudes are characterized by a single parameter, the constant (Arrow–Pratt) coefficient of absolute risk aversion. For such cases the Bernoulli utility index may be expressed as a member of the following family of functions.

Definition 4. A (Bernoulli) utility index $v : \mathbb{R} \to \mathbb{R}$ has the *constant absolute risk aversion (CARA)* form with parameter α if it can be expressed as a positive affine transformation of the canonical CARA function

$$u(c) = \begin{cases} [1 - \exp(-\alpha c)]/\alpha & \text{if } \alpha \neq 0 \\ c & \text{if } \alpha = 0. \end{cases}$$

Notice that, for the canonical CARA function, and for all α, $u(0) = 0$ and $u'(0) = 1$. Further, if $u(\cdot)$ has the CARA form with parameter α, then the coefficient of absolute risk aversion given by $-u''(c)/u'(c)$ is indeed the same for all c and equal to α.

For ease of exposition, for $i = \text{A}, \text{P}$, we use $-i$ to denote the party who is not i. Furthermore, in order to avoid overly cumbersome subscripts, for any $A \subseteq S^i$, we will abuse our notation and identify $f_A f'$ with $f_{A \times S^{\text{P}}} f'$ when $i = \text{A}$, and with $f_{S^{\text{A}} \times A} f'$ when $i = \text{P}$. Similarly, for any $B \subseteq S^{-i}$, we identify $f_B f'$ with the random variable $f_{S^{\text{A}} \times B} f'$ when $i = \text{A}$, and with $f_{B \times S^{\text{P}}} f'$ when $i = \text{P}$.

Axiom 1 *(Own-Signal Sure Thing Principle [OS-STP]).* For any $A \subseteq S^i$, and any pair of random variables f, f' in $\mathcal{F}$: $f \succsim^i f'_A f \Rightarrow f_A f' \succsim^i f'$.

The own-signal sure thing principle (OS-STP) entails that the preference relation is *additively separable* with respect to her own signal S^i. Furthermore, it allows us to view the individual's *conditional* (on her own-signal realization) preferences over

[13] Skiadas (2013) introduces and axiomatizes a two-stage recursive constant *relative* risk aversion expected utility model in which a homotheticity property of preferences replaces translation invariance.

random variables that are measurable with respect to the other's signal, as *not* depending on what might have happened if another realization of her own signal had obtained. More formally, for any pair of random variables f^{-i} and $\hat{f}^{-i}$ in $\mathcal{F}^{-i}$, any pair of random variables $\hat{f}$ and $\hat{f}'$ in $\mathcal{F}$, and any own signal realization s in S^i, it follows from OS-STP that $f^{-i}_{\{s\}}\hat{f} \succsim^i \hat{f}^{-i}_{\{s\}}\hat{f}$ implies $f^{-i}_{\{s\}}\hat{f}' \succsim^i \hat{f}^{-i}_{\{s\}}\hat{f}'$.[14]

In light of this, the next property, the conditional sure thing principle (C-STP) can be interpreted as requiring the conditional preferences over random variables measurable with respect to the other signal to be additively separable with respect to the other signal S^{-i}.

Axiom 2 *(Conditional Sure Thing Principle [C-STP]).* For any own-signal realization $s \in S^i$, any set of other-signal realizations $B \subseteq S^{-i}$, and any pair of random variables $f^{-i}, \hat{f}^{-i}$ in $\mathcal{F}^{-i}$:

$$f^{-i} \succsim^i \left(\hat{f}^{-i}_B f^{-i}\right)_{\{s\}} f^{-i} \Rightarrow \left(f^{-i}_B \hat{f}^{-i}\right)_{\{s\}} f^{-i} \succsim^i \hat{f}^{-i}_{\{s\}} f^{-i}.$$

Furthermore, for any $s \in S^i$ such that $\left|S^{-i}_s\right| = 2$, the following 'hexagon' condition also holds: for any six constant random variables $c, c', c'', \hat{c}, \hat{c}', \hat{c}''$ in C, any $s' \in S^{-i}_s$ and any random variable $f^{-i} \in \mathcal{F}^{-i}$;

$$\left(c_{\{s'\}}\hat{c}'\right)_{\{s\}} f^{-i} \sim^i \left(c'_{\{s'\}}\hat{c}\right)_{\{s\}} f^{-i}, \text{and } \left(c''_{\{s'\}}\hat{c}\right)_{\{s\}} f^{-i} \sim^i \left(c'_{\{s'\}}\hat{c}'\right)_{\{s\}} f^{-i} \sim^i \left(c_{\{s'\}}\hat{c}''\right)_{\{s\}} f^{-i}$$
$$\text{implies } \left(c'_{\{s'\}}\hat{c}''\right)_{\{s\}} f^{-i} \sim^i \left(c''_{\{s'\}}\hat{c}'\right)_{\{s\}} f^{-i}.$$

The third property, translation invariance, ensures that there are no wealth effects on an individual's attitudes toward the uncertainty associated with the realization of the own signal or associated with the realization of the other's signal.

Axiom 3 *(Translation Invariance [TI]).* For any pair of random variables $f, f' \in \mathcal{F}$, and any constant random variable $c \in \mathcal{C}$,

$$f \succsim^i f' \text{ implies } f + c \succsim^i f' + c.$$

With these preliminaries in hand, we now establish that these three properties are necessary and sufficient for a continuous certainty equivalent representation to admit an RCARA representation.

Theorem 1 *(Representation result). Suppose $\left|S^i\right| > 2$. For any preference relation for which every realization $s \in S^i$ of her own signal is essential and for which there exists a continuous certainty equivalent representation, the following are equivalent:*

1. *The preference relation $\succsim^i$ satisfies the own-signal sure thing principle, the conditional sure thing principle and translation invariance.*
2. *There exist: a probability density $\mu(\cdot,\cdot)$ on Ω, marginal density $\mu_{S^i}(\cdot)$ on S^i, and for each $s \in S^i$, conditional density $\mu_{S^{-i}}(\cdot|s)$ on S^{-i} with support S^{-i}_s; a coefficient of constant (ex ante) absolute risk aversion α_0; and, for each $s \in S^i$, a coefficient of (ex interim) absolute uncertainty aversion α_s, such that the certainty equivalent representation may be expressed as*

$$e(f) = u_0^{-1}\left(\sum_{s\in S^i}\mu_{S^i}(s) \times u_0 \circ v_s^{-1}\left(\sum_{s'\in S^{-i}_s}\mu_{S^{-i}}\left(s'|s\right) v_s\left(f\left(s,s'\right)\right)\right)\right),$$

where $u_0(\cdot)$ takes a CARA form with parameter α_0,

and for each $s \in S^i$, $v_s(\cdot)$ takes a CARA form with parameter α_s.

Moreover, $\mu(\cdot,\cdot)$ and α_0 are unique, as is α_s for every own-signal realization s for which the conditional density $\mu_{S^{-i}}(\cdot|s)$ is not degenerate (that is, for which $\left|S^{-i}_s\right| > 1$).

Proof. 1. $\Rightarrow$ 2. A sketch of the proof is as follows. Step 1. We establish that the restriction of the preference relation $\succsim^i$ to $\mathcal{F}^i$ admits a (unique) certainty equivalent CARA representation $e_0\left(f^i\right)$. Step 2. For each s in S^i, we define the conditional preference $\succsim^i_{\{s\}}$ on $\mathcal{F}^{-i}$ and show that it inherits the properties of additive separability over S^{-i} and translation invariance. Hence $\succsim^i_{\{s\}}$ admits a unique certainty equivalent CARA representation $e_s\left(f^{-i}\right)$. Step 3. We convert each act $f \in \mathcal{F}$ to an "equivalent" act f^i in $\mathcal{F}^i$ where for each s in S^i, $f^i\left(s,s'\right) = e_s\left(f^{-i}_s\right)$ and $f^{-i}_s\left(\hat{s},s'\right) = f\left(s,s'\right)$, for all $\hat{s} \in S^i$ and all $s' \in S^{-i}$. It follows from the additive separability of $\succsim^i$ across S^i that $f \sim^i f^i$. Step 4. Set $e(f) := e_0\left(f^i\right)$.

[14] To apply the axiom take $A = \{s\}$, $f = f^{-i}_{\{s\}}\hat{f}$ and $f' = \hat{f}^{-i}_{\{s\}}\hat{f}'$.

Step 1. We begin with the restriction of $\succsim^i$ to $\mathcal{F}^i$, the set of random variables measurable with respect to her own signal. The fact that $\left|S^i\right| > 2$, and for each $s \in S^i$, $\{s\} \times S^{-i} \notin \mathcal{N}$, means that OS-STP implies that the restriction of $\succsim^i$ to $\mathcal{F}^i$ is additively separable across S^i. Furthermore, from TI it follows that certainty equivalent of this restriction satisfies $e_0\left(f^i+c\right) = e_0\left(f^i\right)+c$. As is well-known (Arrow, 1971) this means that $e_0 : \mathcal{F}^i \to \mathbb{R}_+$ is *uniquely* defined. That is, there exists a (strictly positive) probability density $\mu_{S^i}(\cdot)$ on S^i and a constant coefficient of absolute risk aversion, $\alpha_0 \in \mathbb{R}$, such that $e_0\left(f^i\right) = u_0^{-1}\left(\sum_{s\in S^i}\mu_{S^i}(s)\, u_0\left(f^i\left(s,s'\right)\right)\right)$, where $u_0(c) = 1-\exp(-\alpha_0 c)/\alpha_0$, if $\alpha_0 \neq 0$ or $u_0(c) = c$ if $\alpha_0 = 0$, represents $\succsim^i$ restricted to $\mathcal{F}^i$.

Step 2. To extend this representation to general f, we first define for each $s \in S^i$ the binary relation $\succsim^i_{\{s\}}$ on $\mathcal{F}^{-i}$ by setting $f^{-i} \succsim^i_{\{s\}} \hat{f}^{-i}$ whenever $f^{-i} \succsim^i \hat{f}^{-i}_{\{s\}} f^{-i}$. The following Lemma states that $\succsim^i_{\{s\}}$ does not depend on what determines the outcome on the complement of $\{s\} \times S^{-i}$.

Lemma 2. *If $f^{-i} \succsim^i_{\{s\}} \hat{f}^{-i}$ then $f^{-i}_{\{s\}}\tilde{f} \succsim^i \hat{f}^{-i}_{\{s\}}\tilde{f}$, for all $\tilde{f} \in \mathcal{F}$.*

Proof. If $f^{-i} \succsim^i_{\{s\}} \hat{f}^{-i}$ then by definition $f^{-i} \succsim^i \hat{f}^{-i}_{\{s\}} f^{-i}$. Thus by applying OS-STP for $A = \{s\}$, $f = f^{-i}$ and $\hat{f} = \hat{f}^{-i}_{\{s\}}\tilde{f}$, we have $f^{-i} = f \succsim \hat{f}_{\{s\}} f = \hat{f}^{-i}_{\{s\}} f^{-i}$, implies $f_{\{s\}}\hat{f} = f^{-i}_{\{s\}}\tilde{f} \succsim \hat{f}^{-i}_{\{s\}}\tilde{f} = \hat{f}$, as required. $\square$

An immediate implication of Lemma 2 is that this derived relation $\succsim^i_{\{s\}}$ inherits the properties of completeness and transitivity from $\succsim^i$. Furthermore, since $\succsim^i$ admits a continuous certainty equivalent representation, it follows that its restriction to random variables that agree outside the event $\{s\} \times S^{-i}$ to some random variable $\tilde{f}$ (and by Lemma 2 to *any* random variable $\tilde{f}$) satisfies continuity with respect to pointwise convergence. Thus there exists a unique function $e_s : \mathcal{F}^{-i} \to \mathbb{R}_+$, that represents $\succsim^i_{\{s\}}$. It remains to show that $\succsim^i_{\{s\}}$ also satisfies TI.

Lemma 3. *For any pair of random variables measurable with respect to the other signal $f^{-i}, \hat{f}^{-i} \in \mathcal{F}^{-i}$, and any constant random variable $c \in \mathcal{C}$,*

$$f^{-i} \succsim^i_{\{s\}} \hat{f}^{-i} \textit{ implies } f^{-i}+c \succsim^i_{\{s\}} \hat{f}^{-i}+c.$$

Proof. If $f^{-i} \succsim^i_{\{s\}} \hat{f}^{-i}$ then by definition $f^{-i} \succsim \hat{f}^{-i}_{\{s\}} f^{-i}$. Applying TI yields

$$\left(f^{-i}+c\right)_{\{s\}}\left(f^{-i}+c\right) \succsim^i \left(\hat{f}^{-i}+c\right)_{\{s\}}\left(f^{-i}+c\right),$$

which by definition gives us $f^{-i}+c \succsim^i_{\{s\}} \hat{f}^{-i}+c$, as required. $\square$

From Lemma 3 it follows that $e_s\left(f^{-i}+c\right) = e_s\left(f^{-i}\right)+c$. Combining this translation invariance with the additive separability across S^{-i} that follows from C-STP, leads to the unique certainty equivalent CARA representation of $\succsim^i_{\{s\}}$ of the form

$$e_s\left(f^{-i}\right) = \begin{cases} v_s^{-1}\left(\sum_{s'\in S_s^{-i}} \mu_{S^{-i}}\left(s'|s\right) v_s\left(f^{-i}\left(s,s'\right)\right)\right) & \text{if } \left|S_s^{-i}\right| > 1 \\ f^{-i}\left(s,\hat{s}'\right), \text{where } \left\{\hat{s}'\right\} = S_s^{-i} & \text{if } \left|S_s^{-i}\right| = 1 \end{cases}$$

Step 3. To complete the proof that the axioms are sufficient, fix a random variable $f \in \mathcal{F}$. For each realization $s \in S^i$, denote by f_s^{-i} the random variable in $\mathcal{F}^{-i}$ in which $f_s^{-i}\left(\hat{s},s'\right) = f\left(s,s'\right)$ for each $\left(\hat{s},s'\right) \in S^i \times S^{-i}$. It follows from OS-STP and C-STP that for each $s \in S^i$, $\left[e_s\left(f_s^{-i}\right)\right]_{\{s\}} f \sim^i f$ and hence that $f \sim^i_{\{s\}} f^i$, where $f^i \in \mathcal{F}^i$ is the random variable (measurable with respect to her own signal) in which $f^i\left(s,s'\right) = e_s\left(f_s^{-i}\right)$ for each $\left(s,s'\right) \in S^i \times S^{-i}$.

Step 4. Setting $e(f) := e_0\left(f^i\right)$ yields the expression in statement 2 of the theorem. $\square$

The necessity of the axioms for the representation is straightforward and so we omit the proof.

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